Generalized Autoregressive Score (GAS) or Dynamic Scoring models (Dysco) models

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Outline I

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- Siem Jan Koopman, Jurgen Doornik, Andrew C. Harvey, and to Sebastien Laurent for incorporating these models into G@RCH.
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- http://www.timberlake-consultants.com
- Teresa and David Corbett
- Noelia Germino

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Independent developers

- Andrew Harvey Professor at Cambridge University

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Model developers

- Siem Jan Koopman (Vrie University of Amsterdam and Tinbergen Institute) giving a lecture at the Royal Statistical Society in London

GAS (Dysco) models

- 2008 Creal, Koopman, and Lucas developed a new class of observation driven models, called GAS models.
- Harvey and Tirthankar Chakravarty (2008): "Beta-t-(E)GARCH" developed the model called it a dynamic scoring model

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Observation driven vs parameter driven models

- 2008 Creal, Koopman, and Lucas developed a new class of observation driven models, called GAS models.
- This is an observation driven model with a scaled score function and hence the name—generalized autoregressive score model.
- Harvey and Chikratarvey (2008) independently developed the model and called it a dynamic scoring model.
- Parameter driven models are stochastic models that have their own source of error.

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Observation driven models are a new class of models

- Includes time varying parameters in the models.
- models include non-linear regression models with time-varying parameters.
- include multivariate point process models with time varying parameters.
- includes models for time-varying higher moments.
- includes multivariate models.
- can include some dynamic factor models
- include Engle's ARCH, ACD, ACI, ACM, and DCC models

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Rob Engle NYU Stern Finance

- Rob Engle Noble Laureate author of Autoregressive Conditional Heteroscedastic, Autoregressive Conditional Duration, Autoregressive Conditional Intensity, Autoregressive Conditional Multinomial and Dynamic Conditional Correlation models

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Basic model specification

- Let y_t be a dependent variable of interest,
- Let f_t be a time-varying parameter,
- Let x_t = a vector of exogenous variables, all at time t, and
- Let θ = a vector of static parameters.
- Assume that observation y_t is generated by observation density

$$p(y_t|f_{t-1}, Y^{t-1}, X_1^t, F_1^{t-2}; \theta) \tag{1}$$

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Basic model specification 2

- So the updating equation is:

$$f_t = \omega + \sum_{i=0}^{p-1} A_i s_{t-i} + \sum_{j=1}^{q} B_j f_{t-j} \tag{2}$$

- s_t = some scaled score function.
- f_t would have to evolve subject to its own independent source of error.
- unknown coefficients in $\omega, A_i,$ *and* B_j are functions of θ as well.

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Estimation by simulation

- Simulations are often effective when based on second-order expansions of the log observation density.

$$s_t = S_{t-1} * \bigtriangledown t \tag{3}$$

where $\bigtriangledown$ = score or gradient = $\frac{\partial ln(p)}{\partial f_{t-1}}$ and S_{t-1} = pseudo-inverse information matrix: $= I_{t-1}^{-1} = E_{t-1}[\bigtriangledown t \bigtriangledown_t']^{-1}$

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Gas model specification

GAS models are speccified by the orders of A and B parameters- e.g.

$$GAS(p, q) \tag{4}$$

- where p=order of A_i and q = order of B_j
- If we let $y_t = \sigma_{t-1}\epsilon_t$.
- Therefore a GAS(1,1) model is equivalent to a GARCH(1,1) model in specification.

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EGAS model for female Anxiety

with GARCH(1,1) specification

That is, because Rob Engle defined $\epsilon_t = \sigma_t z_t \quad z_t \sim N(0,1)$, this GAS model is formulated as:

$$\sigma_t^2 = \omega + \alpha_1 z_{t-1}^2 \sigma_{t-1}^2 + \phi_1 \sigma_{t-1}^2 \tag{5}$$

and the Exponential GAS model is formulated as

$$ln\sigma_t^2 = \omega + \alpha_1 u_{t-1} \sigma_{t-1}^2 + \phi_1 ln\sigma_{t-1}^2 \tag{6}$$

where $u_{t-1} = z_t^2 - 1$ *when* $z_t \sim N(0,1)$.

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EGAS model for female Anxiety

with GARCH(1,1) specification

Our mean model for female anxiety is

$$femanx = .242 + \epsilon_t \tag{7}$$

and our EGAS model is

$$ln\sigma_t^2 = -.906 + 0.377\sigma_{t-1}^2 + 0.874 ln\sigma_{t-1}^2 \tag{8}$$

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EGAS diagnostics for female Anxiety model

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TESTS :
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Information Criteria (to be minimized)
Akaike         -2.594384  Shibata        -2.628784
Schwarz        -2.404069  Hannan-Quinn   -2.536203
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Normality Test

                    Statistic       t-Test      P-Value
Skewness             -0.23534      0.53422      0.59319
Excess Kurtosis   -0.0062462    0.0072772      0.99419
Jarque-Bera          0.25850          .NaN      0.87875
---------------

Diagnostic test based on the news impact curve (EGARCH vs. GARCH)
                                    Test  P-value
Sign Bias t-Test                 2.01201  0.04422
Negative Size Bias t-Test        2.18704  0.02874
Positive Size Bias t-Test        0.96190  0.33610
Joint Test for the Three Effects 7.65660  0.05367

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ARCH 1-2 test:    F(2,21)   =  0.20989 [0.8124]
ARCH 1-5 test:    F(5,15)   =  0.96270 [0.4709]
ARCH 1-10 test:   F(10,5)   =  0.74825 [0.6752]
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Joint Statistic of the Nyblom test of stability: 1.79629

Individual Nyblom Statistics:
Cst(M)           0.73528
Cst(V)           0.28970
GAS(Alpha1)      0.44980
GAS(Phi1)        0.26934

Rem: Asymptotic 1% critical value for individual statistics = 0.75.
     Asymptotic 5% critical value for individual statistics = 0.47.
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Adjusted Pearson Chi-square Goodness-of-fit test

# Cells(g)  Statistic      P-Value(g-1)     P-Value(g-k-1)
    40       34.8571         0.659297         0.474996
    50       36.2857         0.911016         0.819660
    60       57.7143         0.523005         0.375235
Rem.: k = 4 = # estimated parameters
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Figure 4: Diagnostics for EGAS female anxiety model

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EGAS Graphics for female Anxiety model

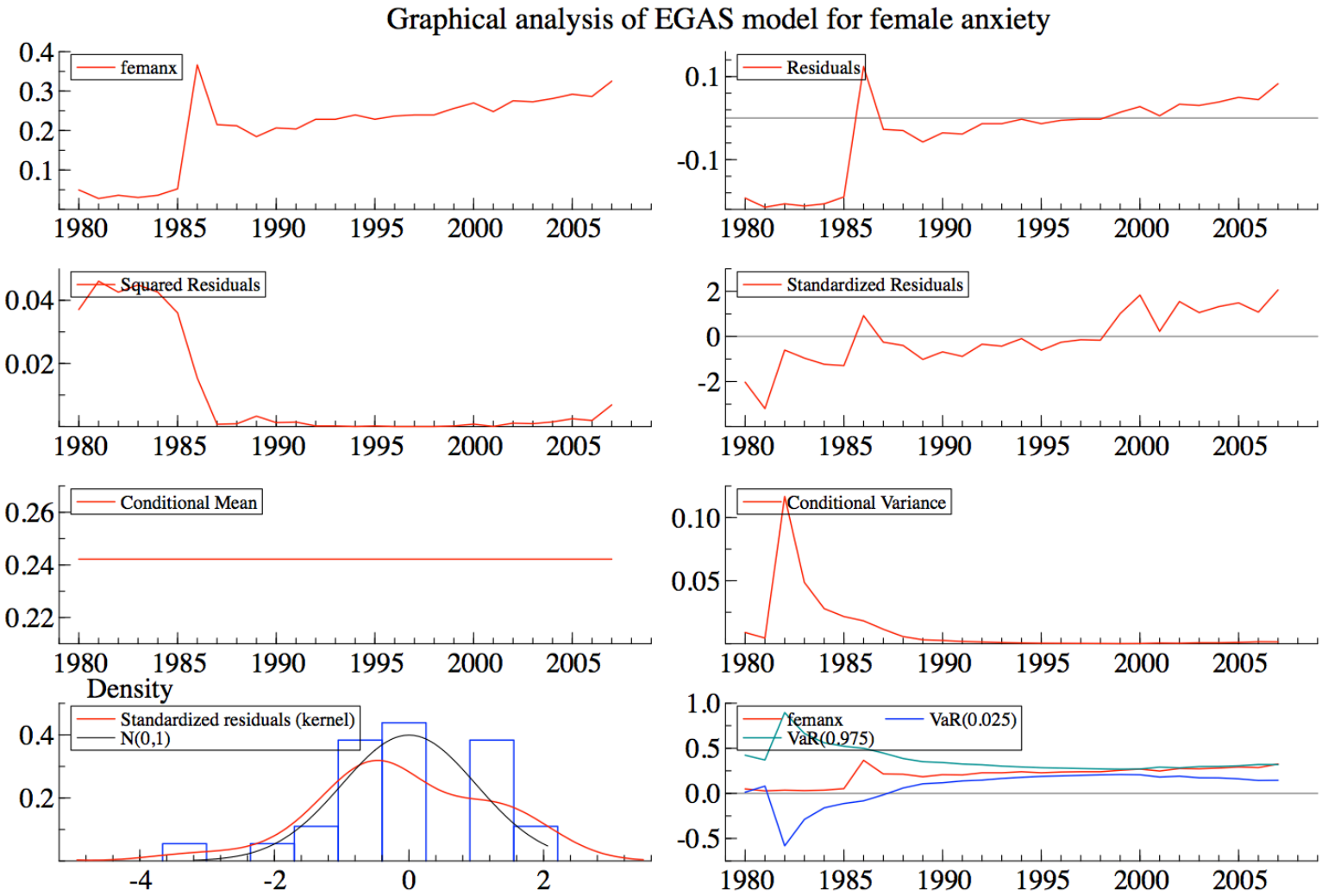

Figure 5: EGAS graphics for female anxiety model

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EGAS forecasts

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***************
** FORECASTS **
***************
Number of Forecasts: 30

Horizon       Mean   Variance
      1     0.2422   0.004994
      2     0.2422   0.003928
      3     0.2422   0.003184
      4     0.2422    0.00265
      5     0.2422   0.002257
      6     0.2422   0.001961
      7     0.2422   0.001735
      8     0.2422   0.001558
      9     0.2422   0.001419
     10     0.2422   0.001307
     11     0.2422   0.001217
     12     0.2422   0.001143
     13     0.2422   0.001082
     14     0.2422   0.001031
     15     0.2422  0.0009887
     16     0.2422  0.0009531
     17     0.2422   0.000923
     18     0.2422  0.0008975
     19     0.2422  0.0008758
     20     0.2422  0.0008572
     21     0.2422  0.0008413
     22     0.2422  0.0008277
     23     0.2422  0.0008159
     24     0.2422  0.0008057
     25     0.2422   0.000797
     26     0.2422  0.0007894
     27     0.2422  0.0007828
     28     0.2422  0.0007771
     29     0.2422  0.0007721
     30     0.2422  0.0007678
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** VaR FORECASTS **
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Number of Forecasts: 30

Horizon       0.05       0.95
      1      0.126     0.3584
      2     0.1391     0.3453
      3     0.1494      0.335
      4     0.1575     0.3269
      5     0.1641     0.3203
      6     0.1693      0.315
      7     0.1737     0.3107
```

Figure 6: EGAS forecasts for female anxiety model