An Introduction to New Developments in OxMetrics

Regime Switching Models

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Outline I

1. Markov-Switching Mean Models
   - Markov-Switching mean models: Dynamic Regression models
   - Markov-Switching: AR models
     - Caveats of MS-AR Models
   - Markov-Switching models with heteroscedasticity
   - Markov-Switching models with component structure
     - variance type: mean-variance component
2. Estimation of Markov-Switching Models
   - nonlinear programming
   - Maximum likelihood
3. Generalized Impulse Response Analysis
4. Forecasting
   - from static models
   - from MS-AR models
5. Markov-Switching Volatility Models
   - Markov-Switching GARCH

Outline II

- Markov-Switching Multi-Fractal Volatility

How do Regime Switching mean models differ from conventional Dynamic Regressions?

The Constant becomes a random variable

$$y_t = \nu + \beta_1 y_{t-1} + X_t' + \epsilon_t, \quad \epsilon_t \sim IIN(0, \sigma_t^2) \tag{1}$$

$$y_t = S(t) + \beta_1 y_{t-1} + X_t' + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_t^2) \tag{2}$$

$$s.t.$$

$$S(t) = c_0 \ if \ regime == 0,$$

$$S(t) = c_1 \ if \ regime == 1$$

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S(t) can be interpreted as an unobserved state or regime

$$p(i|j) = S_{t+1} = i|S_t = j \; for \; i, j = 0, ..., S_{t-1} \tag{3}$$

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$$p(i|j) = S_{t+1} = i|S_t = j \; for \; i,j = 0, ..., S_{t-1} \tag{3}$$

- Because the system has to be in one of S states,

Sum of States

$$\sum_{i=0}^{S-1} p(i|j) = 1 \tag{4}$$

Open dialog Box

- Select Category: models for time series data for now

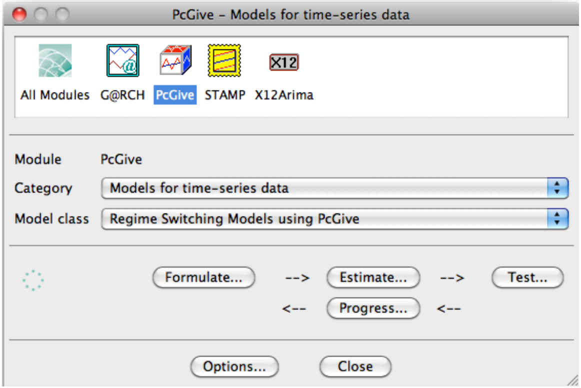
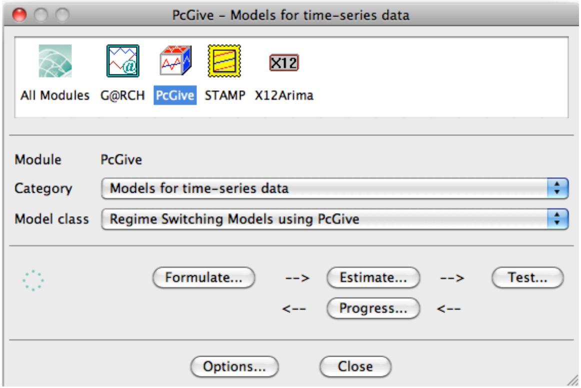


Figure 1: *Select Model class: Regime switching models*

Open dialog Box

- Select Category: models for time series data for now
- We will iterate through estimate, test, and progress later



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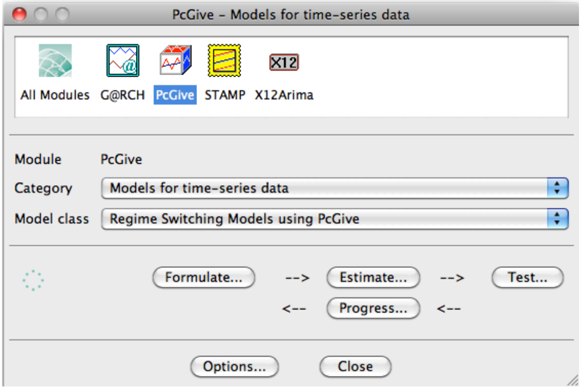
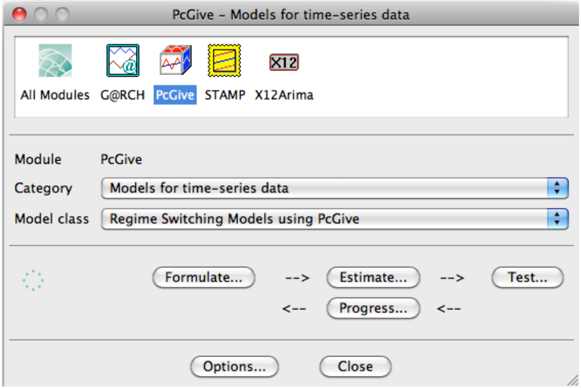


Open dialog Box

- Select Category: models for time series data
- Double click on formulate

Figure 2: *Select Model class: Regime switching models*



Designating the series and the basis of the regime

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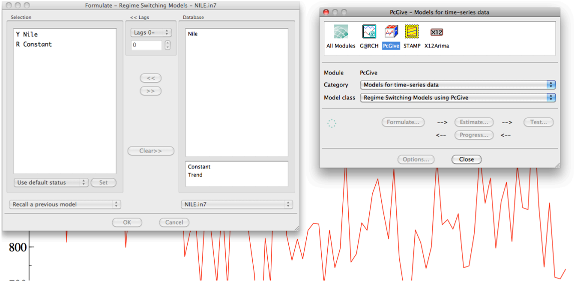

Figure 3: *formulation selection of Nile and constant, and then OK*

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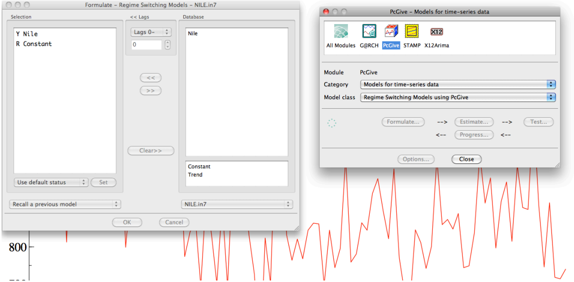
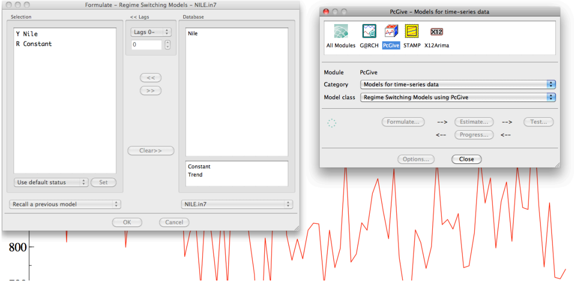


Figure 4: *formulation selection of Nile and constant, and then OK*

Designating the series and the basis of the regime

- Select Category: models for time series data
- Select the Nile time series along with the restricted constant and click on OK



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Type and Number of regimes

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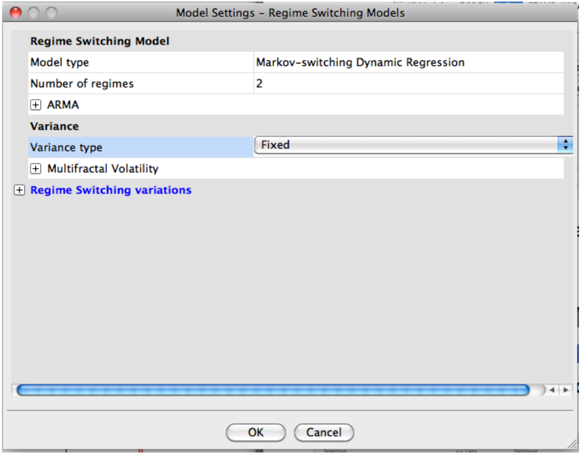

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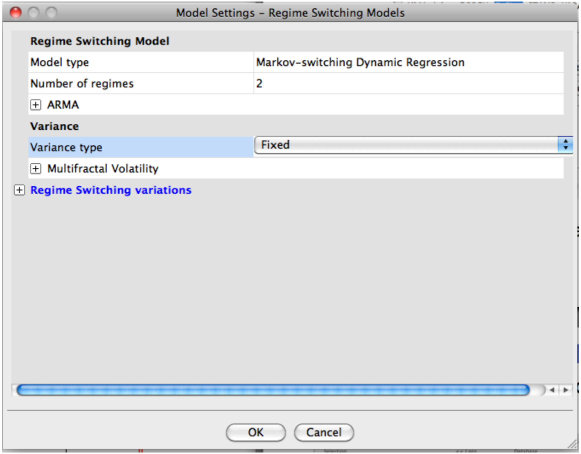

Type and Number of regimes

- We use the type: Markov-Swiching Dynamic Regression
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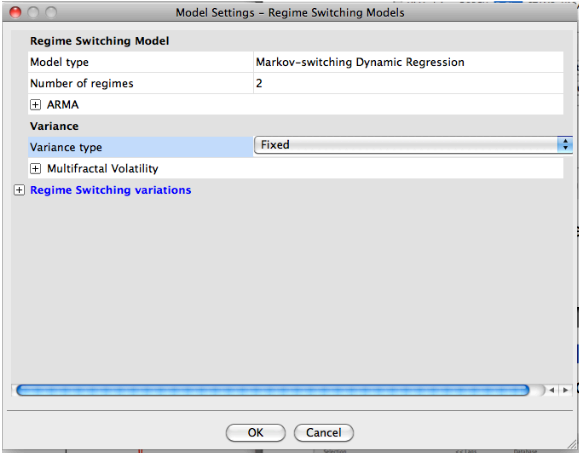
- We allow the variance of the regimes at fixed and click OK



Sample selection

- We use the defaults here and click OK

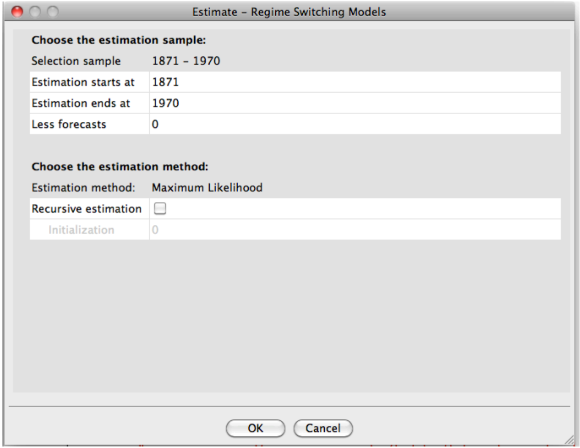


Figure 6: *Using the defaults, and then click OK*

Model selection

- Selecting model in the upper left graphics options yields colored representation different smoothed means.

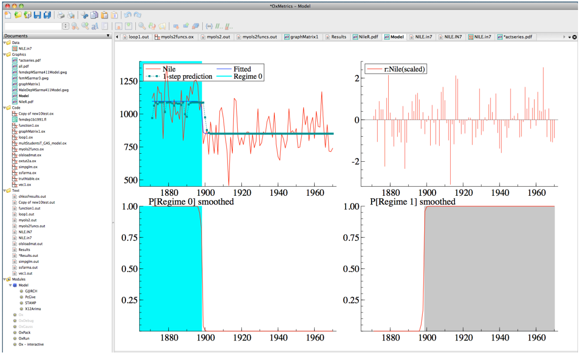


Figure 7: *Graphics model selection reveals building of Aswan Dam in 1899 as change point of regime*

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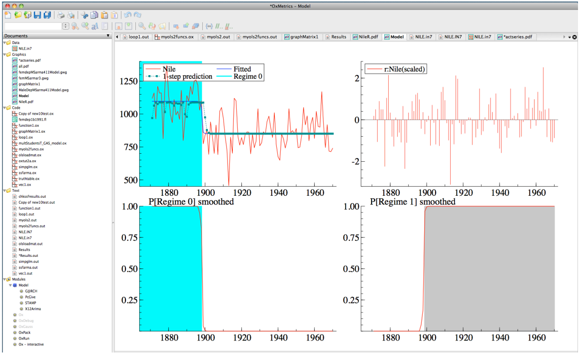

Figure 8: *Graphics model selection reveals building of Aswan Dam in 1899 as change point of regime*

After the level shifting constant, you obtain the transition probability matrix

Effects originate at the top of the matrix and then proceed downward and to the left:

$$\begin{matrix} & \downarrow \\ \leftarrow & \end{matrix}$$

$$P = \begin{pmatrix} & \underline{S_t = 0} & \underline{S_t = 1} \\ S_{t=1} = 0| & p(0|0) & p(0|1) \\ \underline{S_{t+1} = 1|} & \underline{p(1|0)} & \underline{p(1|1)} \\ \sum & 1 & 1 \end{pmatrix} \tag{5}$$

$$P = \begin{pmatrix} & \frac{S_t = 0}{p(0|0)} & \frac{S_t = 1}{1 - p(1|1)} \\ S_{t=1} = 0| & & \\ S_{t+1} = 1| & 1 - p(0|0) & p(1|1) \\ \hline \sum & 1 & 1 \end{pmatrix} \tag{6}$$

Output from Model selection

reveals the decline in average flow volume from 1097 to 846 m^3/sec.

- If we go to the output section in the navigation window on the left and select Results, we obtain

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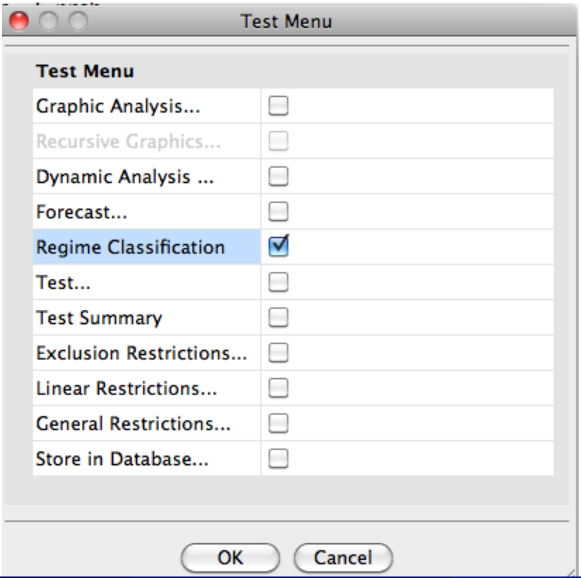
- The level shifts of the constant, their parameter estimates, and the transition probability matrices and settings.

Regime classification selection

Accessing the test menu, we select the regime classification option and click OK



Regime classification selection output

Requesting the regime classification shows the change point of 1899 representing the completion of the Aswan dam.

```
Regime classification based on smoothed probabilities
Regime 0                    years   avg.prob.
          1871 - 1898       28        0.993
  Total: 28 years (31.11%) with average duration of 28.00 years.
Regime 1                    years   avg.prob.
          1899 - 1960       62        0.999
  Total: 62 years (68.89%) with average duration of 62.00 years.
```

- We can use a Likelihood ratio test to compare different models with the df=difference in the number of parameters.

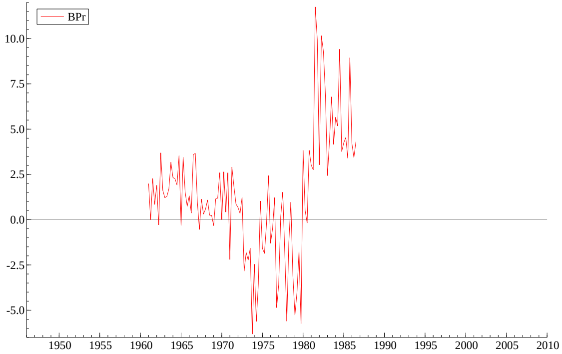


Figure 9: *Bai Perron data on ex post real interest rates*

Modeling with regime switching models

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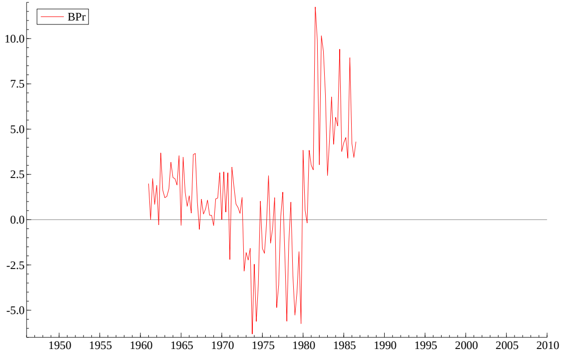
- We use the ex post real interest rates, USIR.in7, file.



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- We load and graph it.

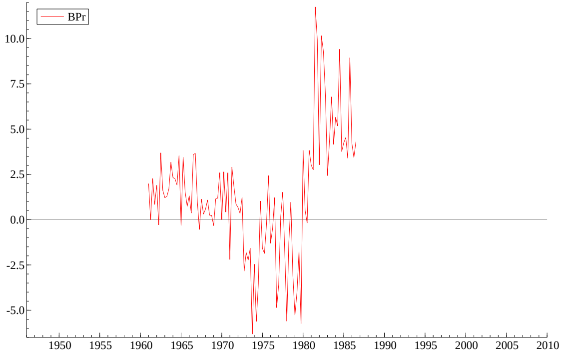

Figure 9: *Bai Perron data on ex post real interest rates*

Bai Perron dialog box settings

- We select the BPir variable

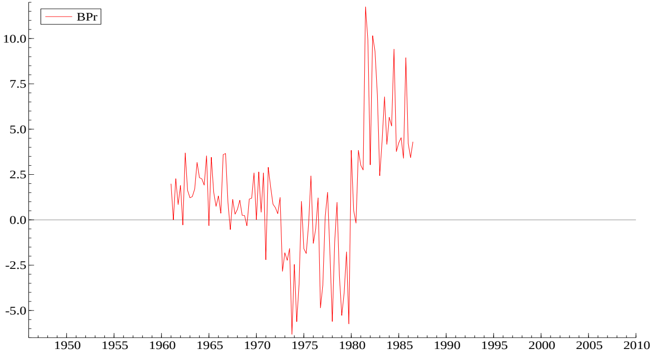

Figure 10: *Bai Perron data on ex post real interest rates*

Bai Perron dialog box settings

- We select the BPir variable
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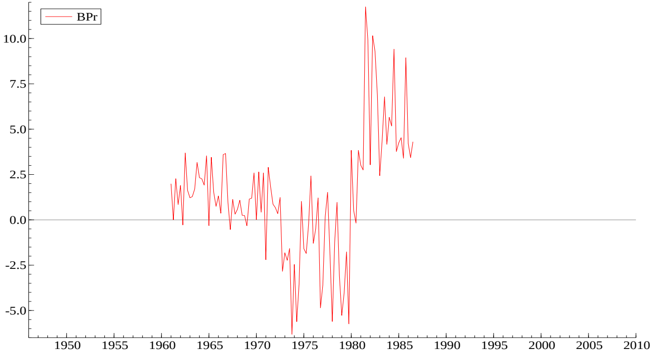

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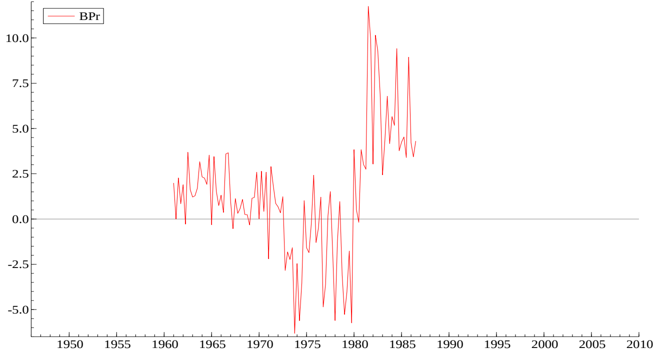

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BP graphical output

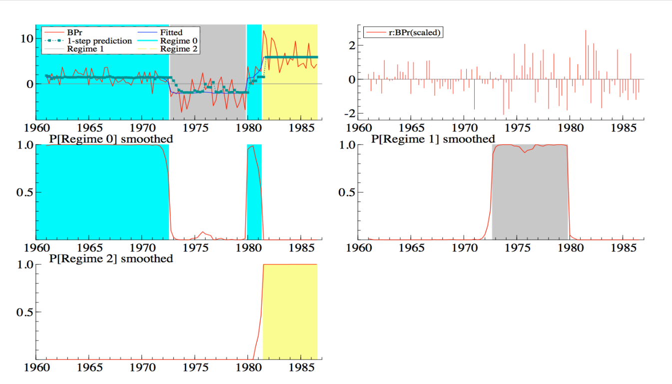


Figure 11: *BP graph output*

Comparing regime switching models

- Select the model dialog box and click on formulate, click OK.

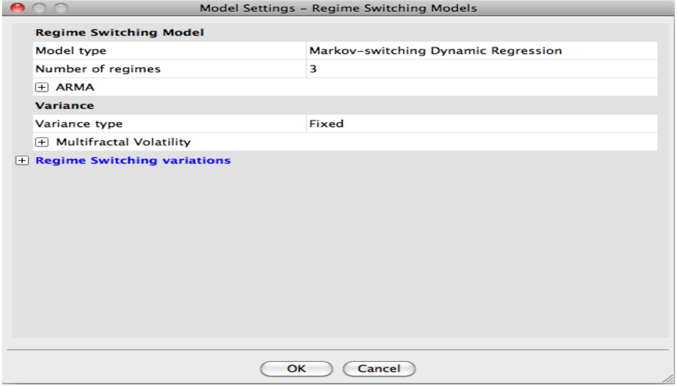
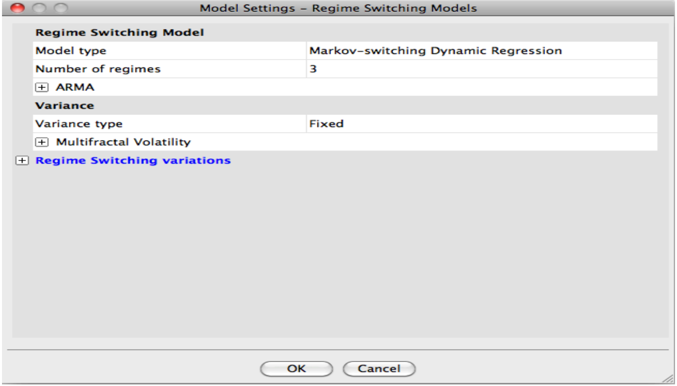


Figure 12: *Select 3 regimes and leave variance type fixed*

Comparing regime switching models

- Select the model dialog box and click on formulate, click OK.
- Enter 3 regimes and allow the variance type to remain fixed, and click OK.



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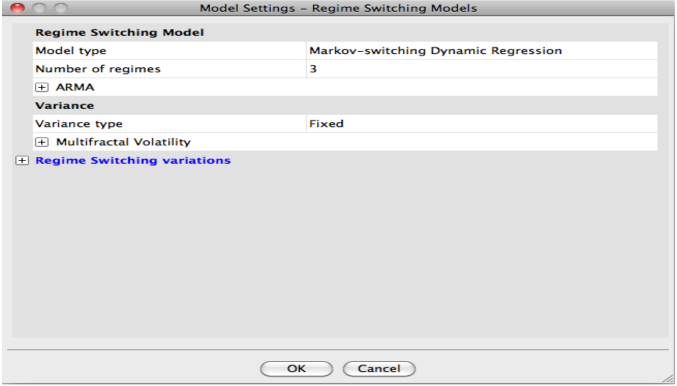
- 

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Formulate the model with three regimes

- Select the model dialog box

```
Switching( 5) Modelling BPr by MS(3)
              The dataset is: /Users/boby/Documents/presentations/OxMetrics2014May16/data/USRIR.in7
              The estimation sample is: 1961(1) - 1986(3)

                Coefficient  Std.Error  t-value  t-prob
Constant(0)         1.41924     0.2904     4.89   0.000
Constant(1)        -2.08089     0.4161    -5.00   0.000
Constant(2)         5.87409     0.4594     12.8   0.000

                Coefficient  Std.Error
sigma               2.03004     0.1470
p_{0|0}            0.959305    0.02831
p_{1|0}           0.0218147    0.02173
p_{0|1}           0.0413714    0.04188

log-likelihood     -231.555696
no. of observations        103  no. of parameters            7
AIC                 4.63214943  SC                  4.81120868
mean(BPr)              1.37514  se(BPr)                3.45123

Linearity LR-test Chi^2(5)  =   83.364 [0.0000]** approximate upperbound: [0.0000]**

Transition probabilities p_{i|j} = P(Regime i at t+1 | Regime j at t)
                Regime 0,t   Regime 1,t   Regime 2,t
Regime 0,t+1       0.95930     0.041371       0.0000
Regime 1,t+1      0.021815      0.95863       0.0000
Regime 2,t+1      0.018881       0.0000       1.0000

Transition probability settings (-1: free parameter, -2: 1-sum(p_{i|.})
                Regime 0,t   Regime 1,t   Regime 2,t
Regime 0,t+1       -1.0000      -1.0000      -2.0000
Regime 1,t+1       -1.0000      -2.0000       0.0000
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Used uniform probabilities to start recursion
Std.Error based on numerical Hessian matrix
SQPF using analytical derivatives (eps1=1e-05; eps2=1e-07):
Strong convergence
Used starting values:
       1.4193      -2.0361       5.8576       2.0408      0.95907     0.020956     0.036736      0.94363
     0.019608      0.96078
```

Figure 13: *Output of three regime interest rates*

Formulate the model with three regimes

- Select the model dialog box
- Double click the progress option

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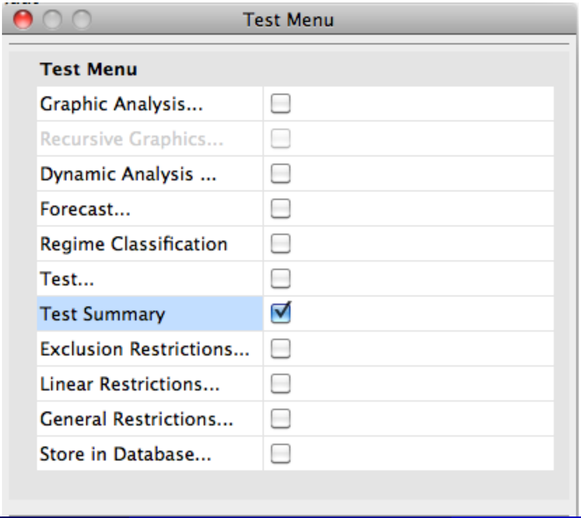
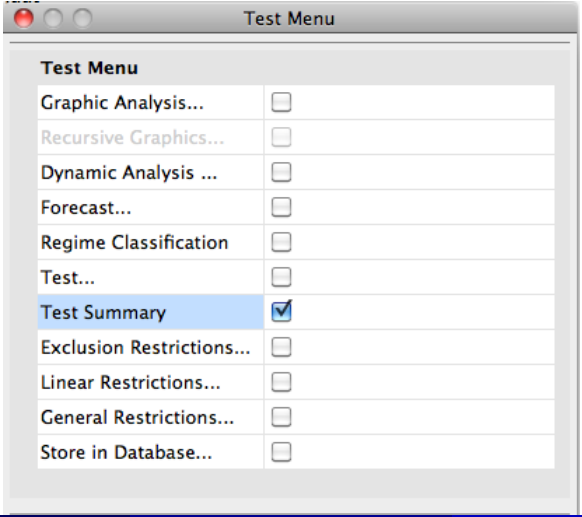
Invoking the Test summary

- The test summary has to be invoked.



Invoking the Test summary

- The test summary has to be invoked.
- It reveals the extent to which the model is properly specified.



Reviewing the diagnostic tests

- The test summary has to be invoked.

```
Descriptive statistics for scaled residuals:
Normality test:    Chi^2(2)  =   2.3007 [0.3165]
ARCH 1-1 test:     F(1,94)   =   1.5096 [0.2223]
Portmanteau(12):   Chi^2(12) =   12.777 [0.3854]
```

Figure 15: *Test summary evaluates model specification*

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Comparison among nested models

- Select the model dialog box

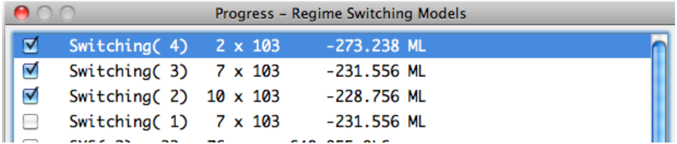

Figure 16: *Selecting the nested models to compare*

Comparison among nested models

- Select the model dialog box
- Double click the progress option

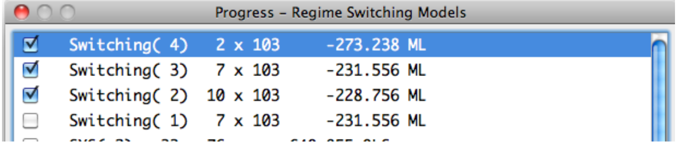

Figure 16: *Selecting the nested models to compare*

Comparison among nested models

- The Model comparison table appears

Progress to date

Model	T	p		log-likelihood	SC	HQ	AIC
S	103	10	M	-228.75575	4.8918	4.7396	4.6360
S	103	7	M	-231.55570	4.8112<	4.7047<	4.6321<
S	103	2	M	-273.23753	5.3956	5.3651	5.3444

Tests of model reduction (please ensure models are nested for test validity)

```
S --> S     : Chi^2( 3) =     5.5999 [0.1328]
S --> S     : Chi^2( 8) =     88.964 [0.0000] **

S --> S     : Chi^2( 5) =     83.364 [0.0000] **
```

Figure 17: *The most parsimonious model is that with 7 parameters*

Optimal model

- The Model comparison table appears

Figure 18: *The most parsimonious model is that with 7 parameters*

The formulation of the MS -AR model

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- Adjustment is a function of the ρ and the number of significant lags.

The MS-AR model

$$y_t - \mu S(t) = \sum_{i=1}^{p} \rho_i (y_i - \mu(S_{t-i})) + \epsilon_t, \quad \epsilon_t \sim IIN(0, \sigma_t^2) \tag{7}$$

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- If the model contains exogenous variables, it is formulated as

The MS-AR model

$$y_t - \mu S(t) - x_t'\gamma = \sum_{i=1}^{p} \rho_i (y_i - \mu(S_{t-i}) - x_t'\gamma) + \epsilon_t, \quad \epsilon_t \sim IIN(0, \sigma_t^2) \tag{8}$$

Caveats for MS-AR models

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- In the MS-AR model that is not so: The Dimension of the state vector is S^{1+p} if estimation is to be performed with ML.
- As Doornik notes on page 24, the slows down MS-AR and renders it infeasible as S and p become large (e.g., where S = 3 and p = 12, N = more than 1 million) [1, 24].

The formulation of the MS -AR model with heteroscedasticity

- Assuming first order autoregression and only two regimes for pedagogical purposes, we obtain

The MS-AR(1) model with heteroscedasticity

$$y_t = \mu(0) + \rho(y_1) + \sigma(0)\epsilon_t \tag{9}$$
$$y_t = \mu(1) + \rho(y_1) + \sigma(1)\epsilon_t \tag{10}$$

Hamilton's MS-AR model

2 regimes and 4 states

The MS-AR model

$$S_t = 0 \; S_{t-1} = 0 \; N_t = 0 \quad (11)$$
$$S_t = 0 \; S_{t-1} = 1 \; N_t = 1 \quad (12)$$
$$S_t = 1 \; S_{t-1} = 0 \; N_t = 2 \quad (13)$$
$$S_t = 1 \; S_{t-1} = 1 \; N_t = 3 \quad (14)$$

- However, this constrains movement.

Hamilton's MS-AR model

2 regimes and 4 states

The MS-AR model

$$S_t = 0 \; S_{t-1} = 0 \; N_t = 0 \qquad (11)$$
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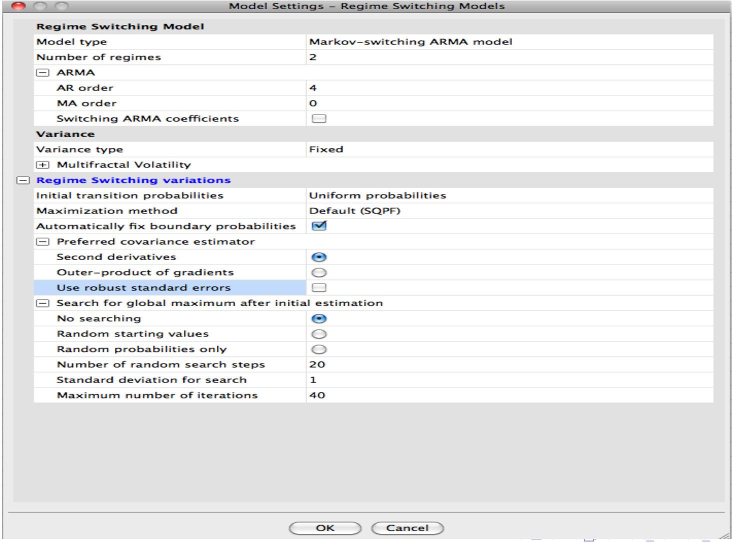
$$S_t = 1 \; S_{t-1} = 1 \; N_t = 3 \qquad (14)$$

- However, this constrains movement.
- We can go from state 0 to state 0 or state 2 but not to the others.

configuring Hamilton's model

- Using OLDDLGDP from USmacro09_q.in7.



Hamilitonian output

- Hamilton's output.

```
Switching(15) Modelling OldDLGNP100 by MS_ARMA(2, 4, 0)
              The dataset is: /Users/boby/Documents/data/OxMetrics2013/data/USmacro09_q.i
              The estimation sample is: 1951(4) - 1984(4)

                  Coefficient   Std.Error   t-value   t-prob
AR-1                0.0623263      0.1479     0.421    0.674
AR-2               -0.0146590      0.1441    -0.102    0.919
AR-3                -0.202860      0.1194     -1.70    0.092
AR-4                -0.162419      0.1172     -1.39    0.168
Constant(0)           1.17225      0.1015      11.5    0.000
Constant(1)         -0.260586      0.2796    -0.932    0.353

                  Coefficient   Std.Error
sigma                0.795379     0.07405
p_{0|0}              0.894837     0.04413
p_{1|1}              0.765925      0.1073

log-likelihood       -179.93277
no. of observations         129  no. of parameters           9
AIC                  2.92919024  SC                 3.12871204
mean(OldDLGNP100)       0.71974  se(OldDLGNP100)       1.05886

Linearity LR-test Chi^2(3)  =    3.0887 [0.3781]   approximate upperbound: [0.8727]

Transition probabilities p_{i|j} = P(Regime i at t+1 | Regime j at t)
                  Regime 0,t   Regime 1,t
Regime 0,t+1         0.89484      0.23407
Regime 1,t+1         0.10516      0.76593
```

Hamilitonian Regime classification and test summary

```
Regime classification based on smoothed probabilities
Regime 0                    quarters   avg.prob.
    1952(4) - 1953(2)          3        0.848
    1954(3) - 1956(3)          9        0.857
    1958(2) - 1960(1)          8        0.925
    1961(1) - 1969(1)         33        0.959
    1971(1) - 1973(2)         10        0.937
    1975(2) - 1979(1)         16        0.929
    1980(4) - 1981(1)          2        0.749
    1983(1) - 1984(4)          8        0.958
  Total: 89 quarters (68.99%) with average duration of 11.12 quarters.
Regime 1                    quarters   avg.prob.
    1953(3) - 1954(2)          4        0.920
    1956(4) - 1958(1)          6        0.846
    1960(2) - 1960(4)          3        0.830
    1969(2) - 1970(4)          7        0.806
    1973(3) - 1975(1)          7        0.873
    1979(2) - 1980(3)          6        0.735
    1981(2) - 1982(4)          7        0.927
  Total: 40 quarters (31.01%) with average duration of 5.71 quarters.


Descriptive statistics for scaled residuals:
Normality test:    Chi^2(2)  =    2.5440 [0.2803]
ARCH 1-5 test:     F(5,110)  =   0.30175 [0.9109]
Portmanteau(12):   Chi^2(12) =    13.438 [0.3380]
```

Figure 21: *Hamilton regime classification and test summary*

Graphing Hamilton’s model

- Hamilton’s predictions vs NBER recessions.

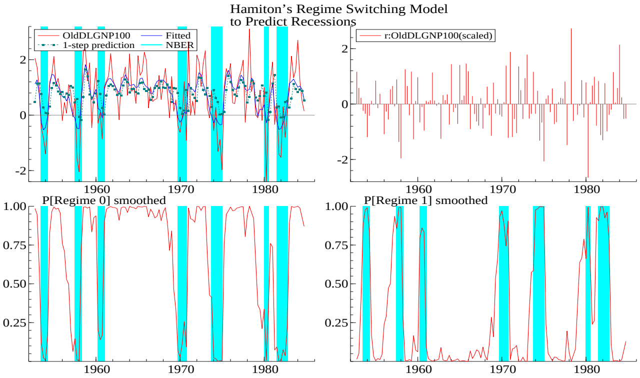


Figure 22: *Hamilton’s recession forecasts v. NBER recessions*

Estimation

- Types of estimation available: Maximum likelihood estimation includes nonlinear programming, EM estimation, BFGS.

Dynamic analysis of the response of the dependent variable over time to a unit impulse at time t

If we modeled the oldDLGNP100 as an MS-AR with 2 regimes and 2 autoregressive lags, we could also examine the impulse response function of the endogenous variable.

The formula for the Impulse response function

$$E(\hat{y}_{T+h}) = E(y_{t+h}|Y_T^{T-p}|X_{T+1}^{T+h}\xi_t, \theta; \epsilon_t, \epsilon_{t+1} \cdots \epsilon_{t-h}) \qquad (15)$$

Select the test menu

Then select the Dynamic Analysis box

- Then click on ok

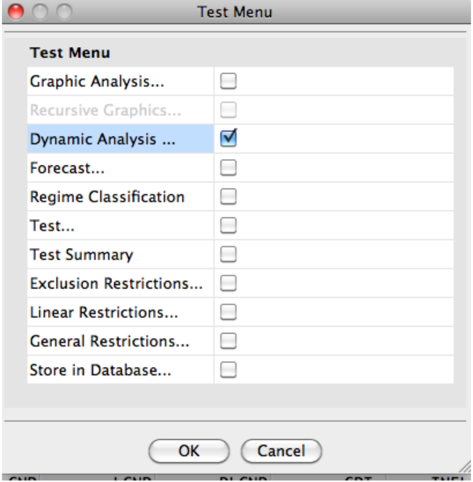


Figure 23: *In the test menu select Dynamic analysis*

Configuring the GIRF

for MS-AR models

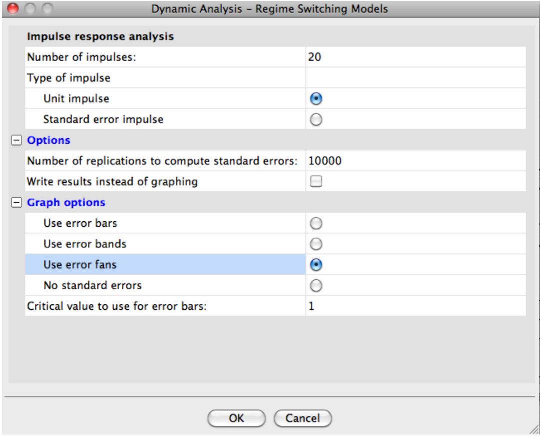


Figure 24: *Select a unit impulse and 20 impulses and error fans*

The Plot of the Generalized IRF
for MS-AR models

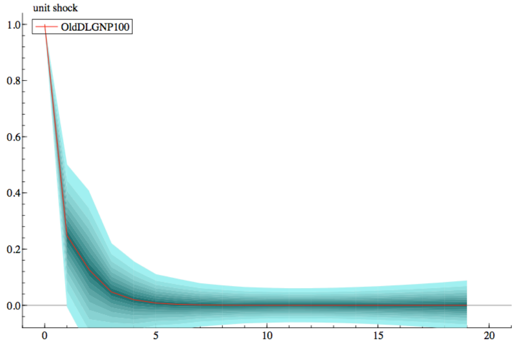


Figure 25: *The Impulse response function plot*

Forecasting

for static models

- Static models with fixed regressors use one-step ahead forecasting.

The first forecast of a one-step ahead forecast is

$$
\begin{aligned}
\hat{y}_{t+1}|Yt^{1} &= E[y_{T+1}|{Y_T}^{1}] \\
&= \textstyle\sum_{j=0}^{S-1} E[y_{T+1}]|S_{T+1} \\
&= j, {Y_T}^{1}|P(S_{T+1} = j_h, {Y_T}^{1}) \\
&= x'_{t+h}\beta_{jh}
\end{aligned} \tag{16}
$$

Forecasting

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&= x'_{t+h}\beta_{jh}
\end{aligned} \qquad (16)
$$

- The forecast is the weighted sum of each of the regimes.

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- The weights are the probabilities of being in that regime.

Forecasting

for static models

The second step of a one-step ahead forecast is

$$\begin{aligned} \hat{y}_{t+2}|T &= \textstyle\sum_{j=0}^{S-1} E[y_{T+2}]|S_{T+2} \\ &= k, {Y_T}^1|P(S_{T+2} = k, {Y_T}^1) \end{aligned} \tag{17}$$

For h steps ahead

$$\begin{aligned}\hat{y}_{t+h}|T &= \sum_{k=0}^{S-1} E[y_{T+h}]|S_{T+h} \\ &= k, {Y_T}^1|P(S_{T+h} = k, {Y_T}^1)\end{aligned} \tag{18}$$

and we can substitute the exogenous parameters so that

$$\hat{y}_{t+h}|T = \sum_{k=0}^{S-1} x_{T+h}\beta_k P(S_{T+h} = k, {Y_T}^1) \tag{19}$$

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Forecasting

for MS-AR models with a regime switching mean

Consider the MS-AR Model

$$\hat{y}_t = \rho y_{t-1} + \mu(S_t) - \rho mu(S_{t-1}) + \epsilon_t \quad \epsilon_t \sim IIN(0, \sigma^2) \tag{20}$$

The first step of a one-step ahead forecast is

$$E(\hat{y}_{t+1}|S_{T+1} = j_1, ..., S_t = j_0 Y_t{}^1) \quad = \rho y_t + \mu(j_1) - \rho\mu(j_0) \tag{21}$$

Forecasting
for MS-AR models

The second step of a one-step ahead forecast is

$$E(\hat{y}_{t+2}|S_{T+2} = j_2, ..., S_t = j_1 Y_t{}^1) \quad = \rho^2 y_{t+1} + \mu(j_2) - \rho\mu(j_1) \quad (22)$$

For h steps ahead the forecast can be formulated as

$$E(\hat{y}_{t+h}|S_{t+h}=j_h, ..., S_t=j_0, Y_1{}^1) \quad = \rho^h y_{t+h} + \mu(j_h) - \rho\mu(j_1) \quad (23)$$

But PcGive approximates the MS-AR by an MS-DR, for an AR(1) model [1, 48-52]

$$\hat{y} = \hat{\rho}(S_T)y_{t-1} + [1 - \hat{\rho}(S(t)]\hat{\mu}(S_t). \quad (24)$$

MS-Volatility Models

- Not only can the level shift be the criterion of regime change, the second moment can be the criterion as well.

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  - switching variance with shared GARCH
  - switching GARCH

Model settings menu

changing parameter from fixed to a form of variance

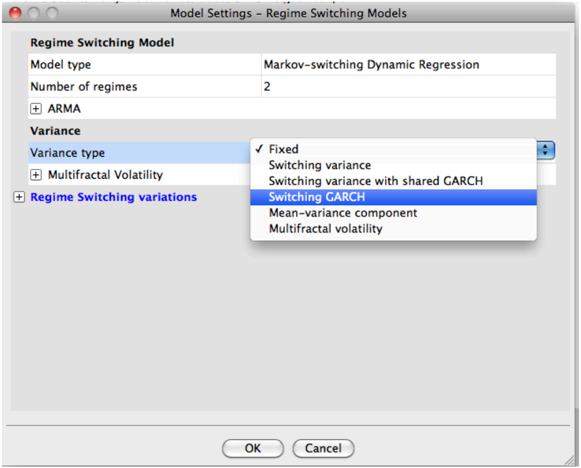

Figure 26: *Variance type options in the model settings menu*

Both MS-Volatility and MS-Multi-fractal Volatility models

MS-Volatility and MSFMV can be expressed as

$$y_t = \sigma(S_t)\epsilon_t \quad \epsilon_t \sim IIN(0,1) \tag{25}$$

state dependent variance is dependent upon baseline value scaled by S volatility components

$$\sigma(S_t)^2 = \sigma_0^2 \Pi_{i=1}^{S} V_{it} \tag{26}$$

Volatility components

- Volatility components are considered to be positive: $V_i > 0$

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- Volatility components are deemed to be independent at any point in time.
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- Therefore, $E[\sigma^2(S_t)] = \sigma_0^2$

Markov-Switching Multi-Fractal Volatility (MSMFV)

- This method of estimation incorporates stochastic volatility components of heterogeneous durations.

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Markov-Switching Multi-Fractal Volatility (MSMFV)

- This method of estimation incorporates stochastic volatility components of heterogeneous durations.
  - captures outliers
  - captures long-memory like volatility persistence
  - captures power variation
- It is applied to compute value at risk, price derivatives, and forecast volatility.

Markov-Switching Multi-Fractal Volatility

Frame of reference

- Let P_t = price of an asset.

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- If r_t = return of the asset = $ln(P_t/P_{t-1})$ over two consecutive periods.

Given the volatility state, M_t, the next period multiplier, $V_{k,t+1}$, is sampled from a fixed distribution, V with probability γ_k, otherwise, it remains the same [2].

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  - where μ and σ are constants and ϵ_t are independent standard Gaussians innovations.

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  - where μ and σ are constants and ϵ_t are independent standard Gaussians innovations.
  - In MSMFV, volatility is a function of $V_t = (V_{1t}, V_{2t}, ..., V_{\bar{k}t})$, which is a latent Markov state vector.

Given the volatility state, M_t, the next period multiplier, $V_{k,t+1}$, is sampled from a fixed distribution, V with probability γ_k, otherwise, it remains the same [2].

Sampling rule

permits estimation by simulated methods of moments

Table 1: *Sampling rule: V_{it} is drawn from distribution f_{it}*

Draw	Probability
$V_{k,t}$ drawn from distribution f_v	with probability γ_k
$V_{k,t} = V_{k,t-1}$	with probability $1 - \gamma_k$

Transition probability

The transition probability

$$\gamma_k = 1 - (1 - \gamma_1)^{b^{k-1}} \tag{27}$$

- At low frequency, the sequence is approximately geometric s. t., $\gamma_k \approx \gamma_1^{k-1}$

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$$\gamma_k = 1 - (1 - \gamma_1)^{b^{k-1}} \tag{27}$$

- At low frequency, the sequence is approximately geometric s. t., $\gamma_k \approx \gamma_1^{k-1}$
- The Marginal distribution, M, has a mean=1 and positive support and is independent of k.

In continuous time Price can be expressed as a diffusion process

$$\frac{dP_t}{P_t} = \mu dt + \sigma(M_t) dW_t \tag{28}$$

References I

[1] Doornik, J. A. (2013) Econometric Analysis with Markov-Switching Models PcGive 14 London, UK: Timberlake Consulting, Ltd., 1-86.