Methods for Modeling and Forecasting Volatility

By
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Presentation at Norwegian University of Science and Technology
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Primary Sources:

- Volatility analysis with G@RCH models (Main source: Estimating and Forecasting ARCH models using G@RCH by Sebastien Laurent)
- Laurent, Sebastien (2007). Estimating and Forecast ARCH Models using G@RCH, Timberlake Consultancy, Ltd. London, UK.
- **G@RCH software by Laurent, S. et. Al. OxMetrics Software, Timberlake Consultancy, Ltd. London, UK. is used owing to its outstanding variety of advanced models and options available at this time.**

Acknowledgments

- I would like to thank Sebastien Laurent for his inspiring teaching and enlightening writing on this subject.
- A considerable amount of inspiration also came from the work of Rob Engle, Jurgen Doornik, and David Hendry as well.
- Also I need to thank Jose Fiuza and Ana Timberlake for their support.
- Finally, Sjur Westgard made this symposium possible and thanks must be given to him.

Outline II

- First generation univariate G@RCH
  - ARCH, GARCH
  - Estimation (QML with bounds and simulated annealing)
  - Diagnostic tests
  - Model comparison
  - Forecasting (Simulated confidence intervals)
  - Forecast Evaluation
  - Simulation of confidence intervals
  - Subset models
  - Outlier modeling
  - Value-at-Risk

Outline III

- Second generation univariate G@RCH
  - Nonstationary GARCH
    - Riskmetrics
    - IGARCH
  - GARCH-in-mean
  - EGARCH
  - GJR GARCH
  - APGARCH
  - Leverage effects and volatility smiles

Outline IV

- Continuous time Models
  - Brownian Motion
  - Integrated and Realized Volatility
  - With Jumps
  - Microstructure noise

- Long-Memory GARCH
  - APARCH
  - FIGARCH
    - FIGARCH- BBM
    - FIGARCH-Chung
    - FIEGARCH
  - FIAPARCH
    - FIAPARCH-BBM
    - FIAPARCH-Chung
  - Davidson’s HYGARCH
  - VaR

Outline IV

– Multivariate G@RCH
- BEKK models
  - Diagonal
  - Scalar
- Factor garch:
  - OGARCH
  - GOGARCH
- Dynamic correlations:
  - Constant Conditional Correlation
  - Dynamic Conditional Correlation

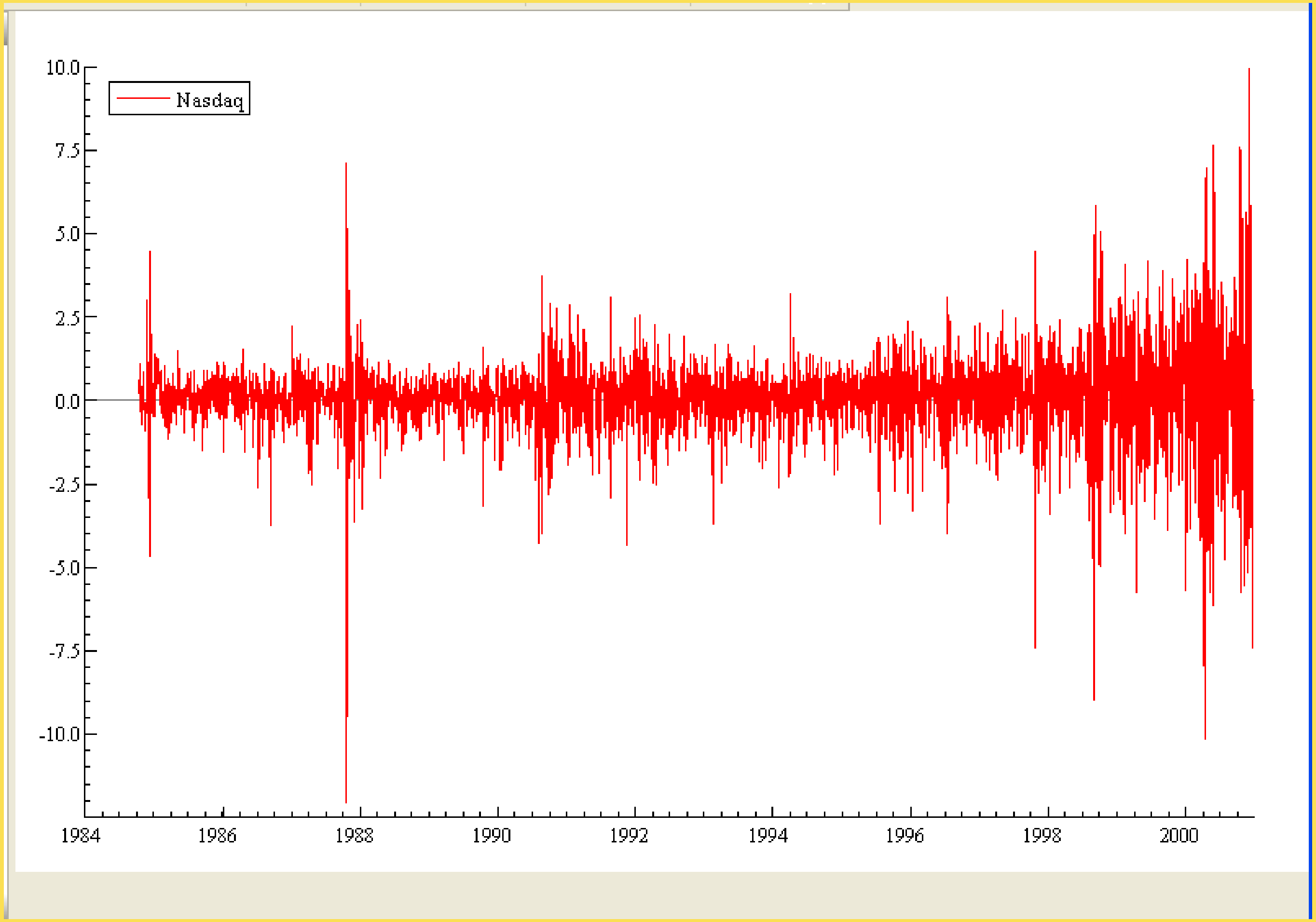
Risk Analysis with G@RCH 5

- We analyze volatility of indicators and assets with G@RCH.
- What is new about G@RCH 5?
    - It contains most of the multivariate Garch models
    - One can obtain the Ox Code for the menu model just run
    - One can model outliers and predictors in the mean and variance models
    - Estimation models has been improved. Simulated annealing option included.
    - Simulation of models is now possible
    - Functions to detect high frequency jumps have been included.

More G@RCH 5 new features

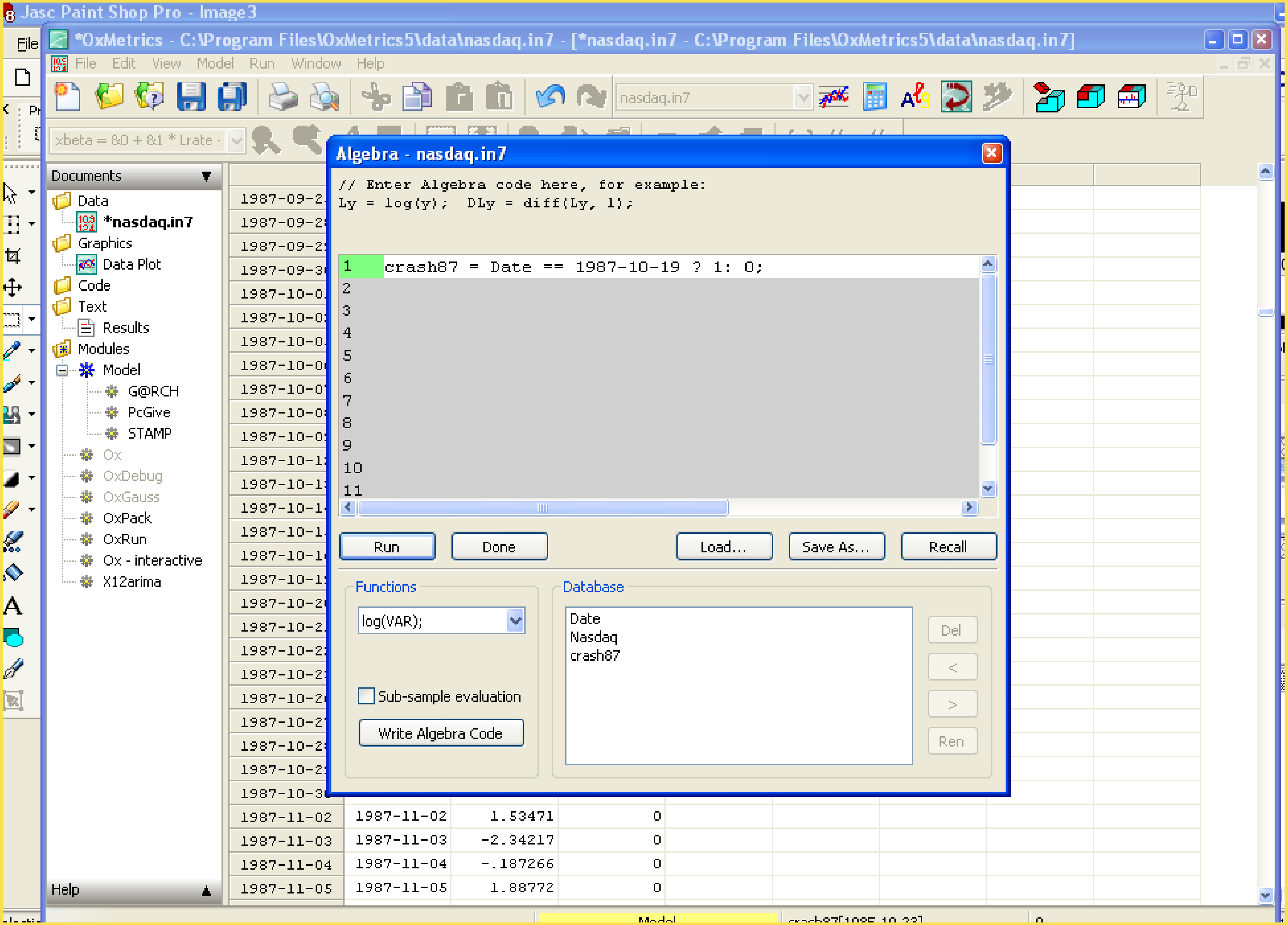
- Simulation capability
- Multivariate GARCH
  - BEKK models
    - Scalar
    - Diagonal
  - Factor GARCH
    - OGARCH
    - GOGARCH
- Conditional Correlations
  - CCC
  - DCC
- Programmable stochastic volatility models

Load and Examine Nasdaq Returns

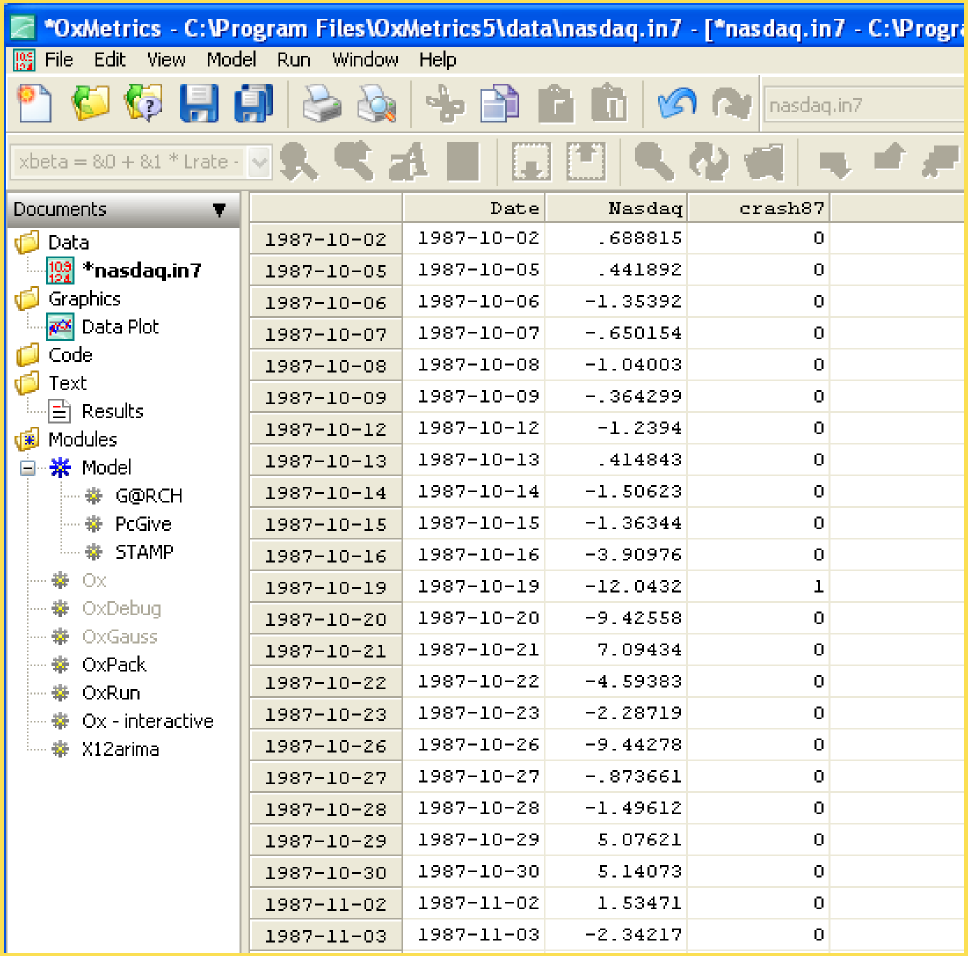


Notice the 1987 crash. We construct a dummy variable for Oct 19, 1987

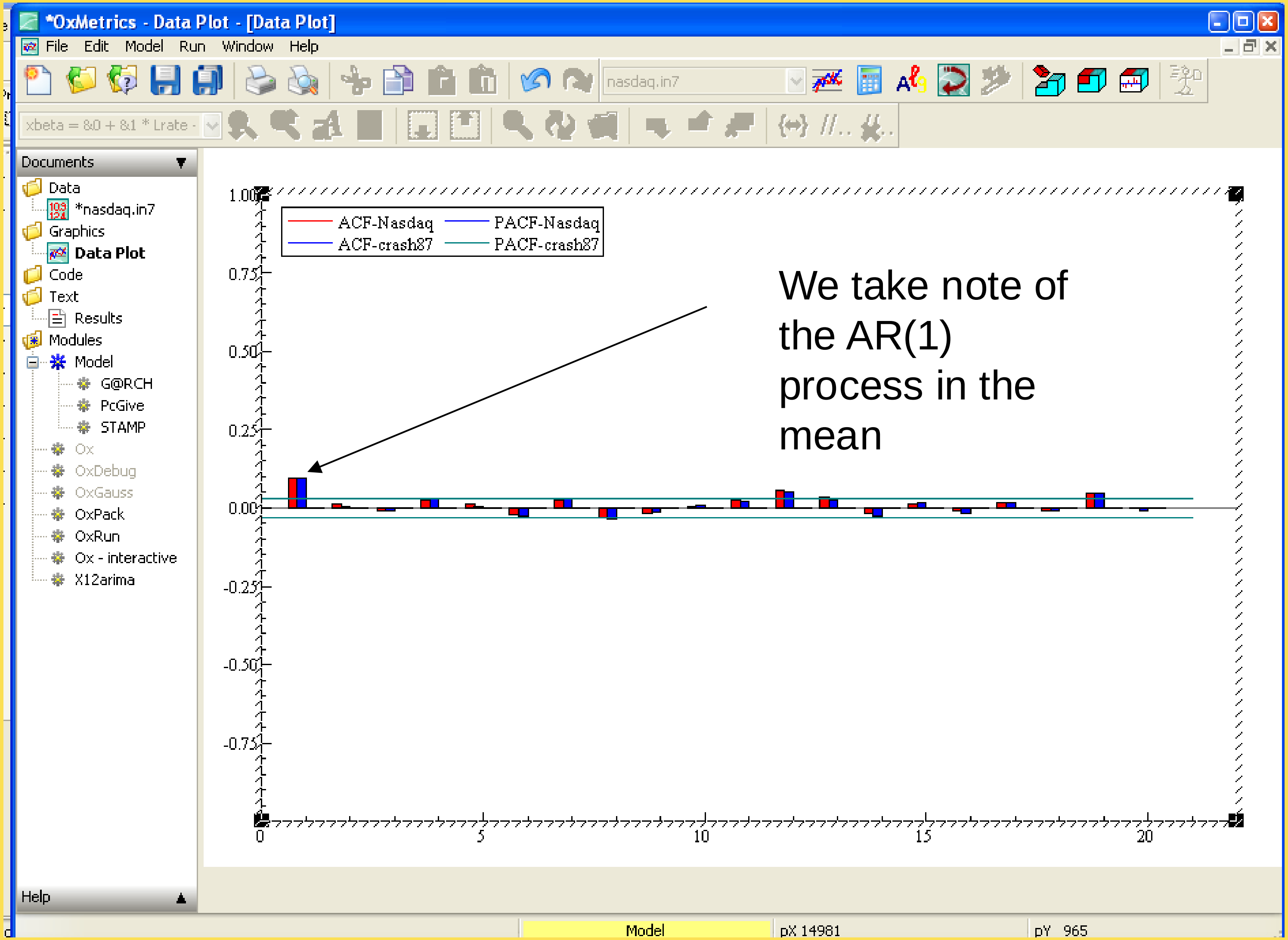
We want record of all variable constructions so I do this with the algebra code



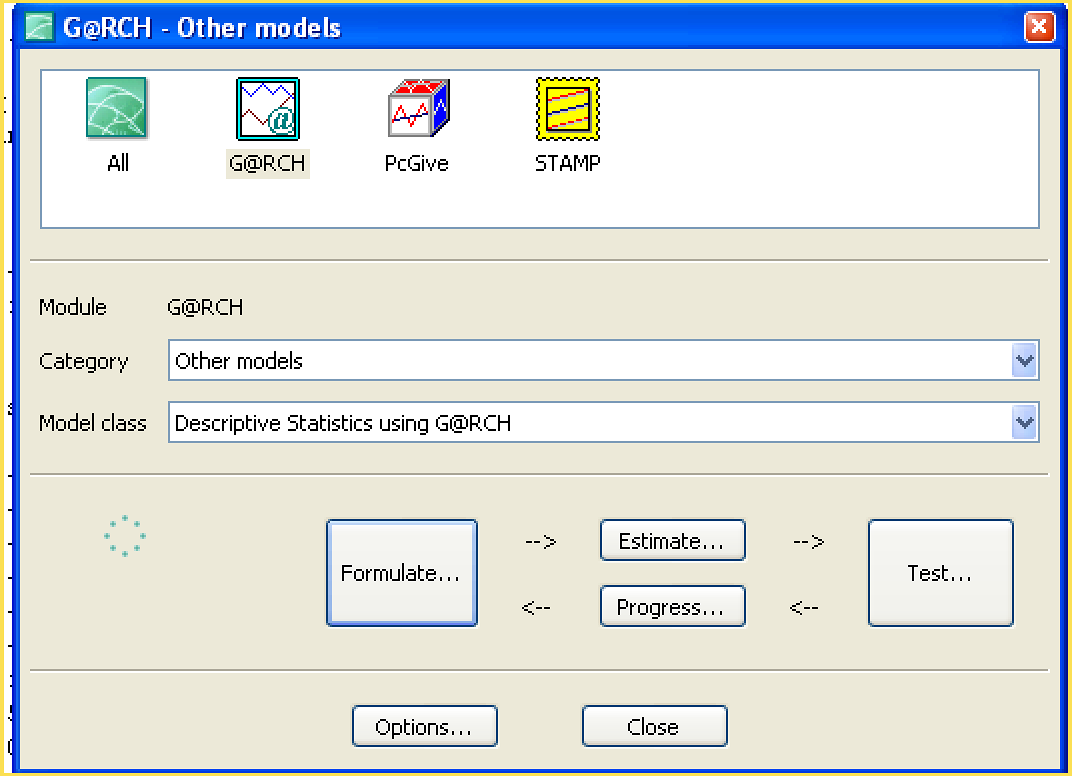
This constructs our dummy variable



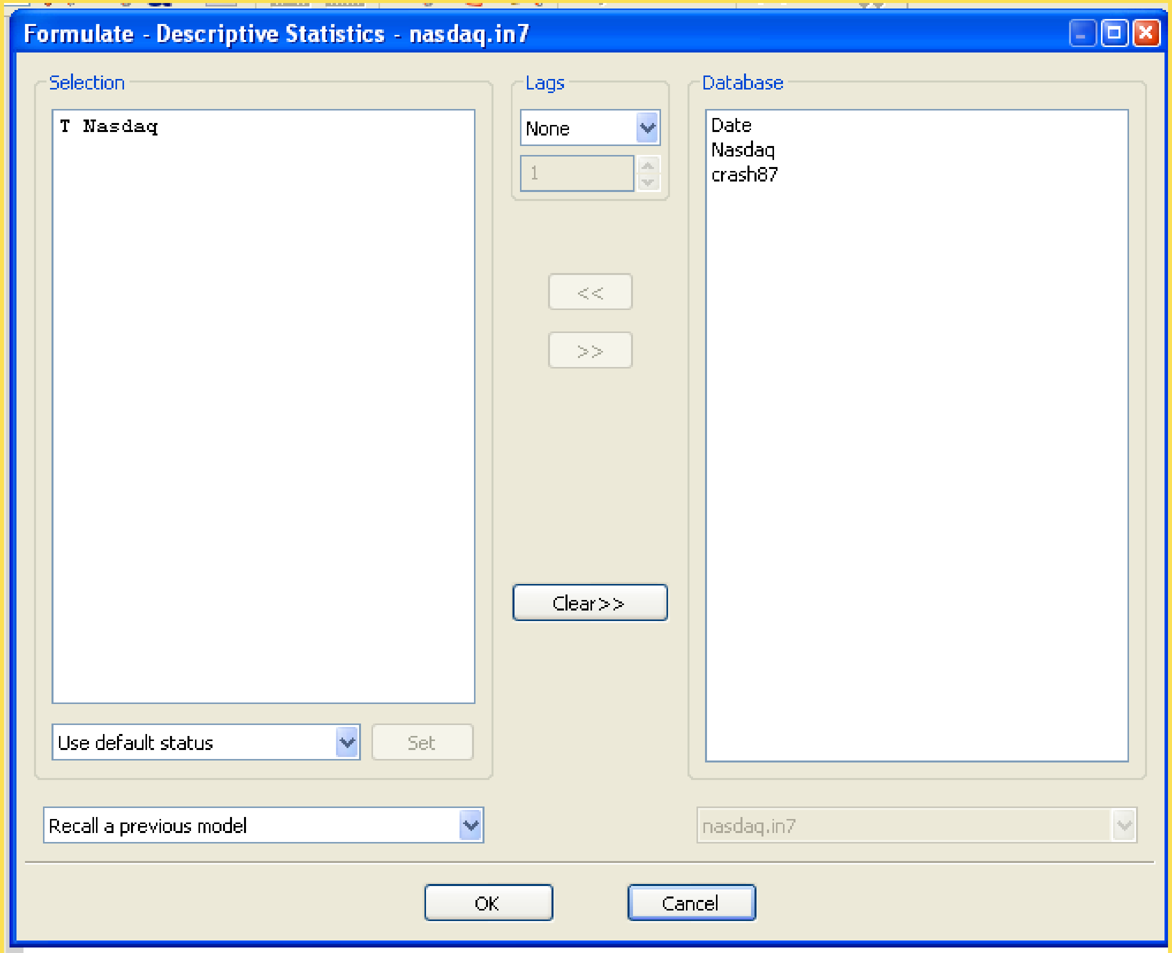
Correlograms reveal an AR(1) and possibly some seasonality



Basic Pre-Model Analysis

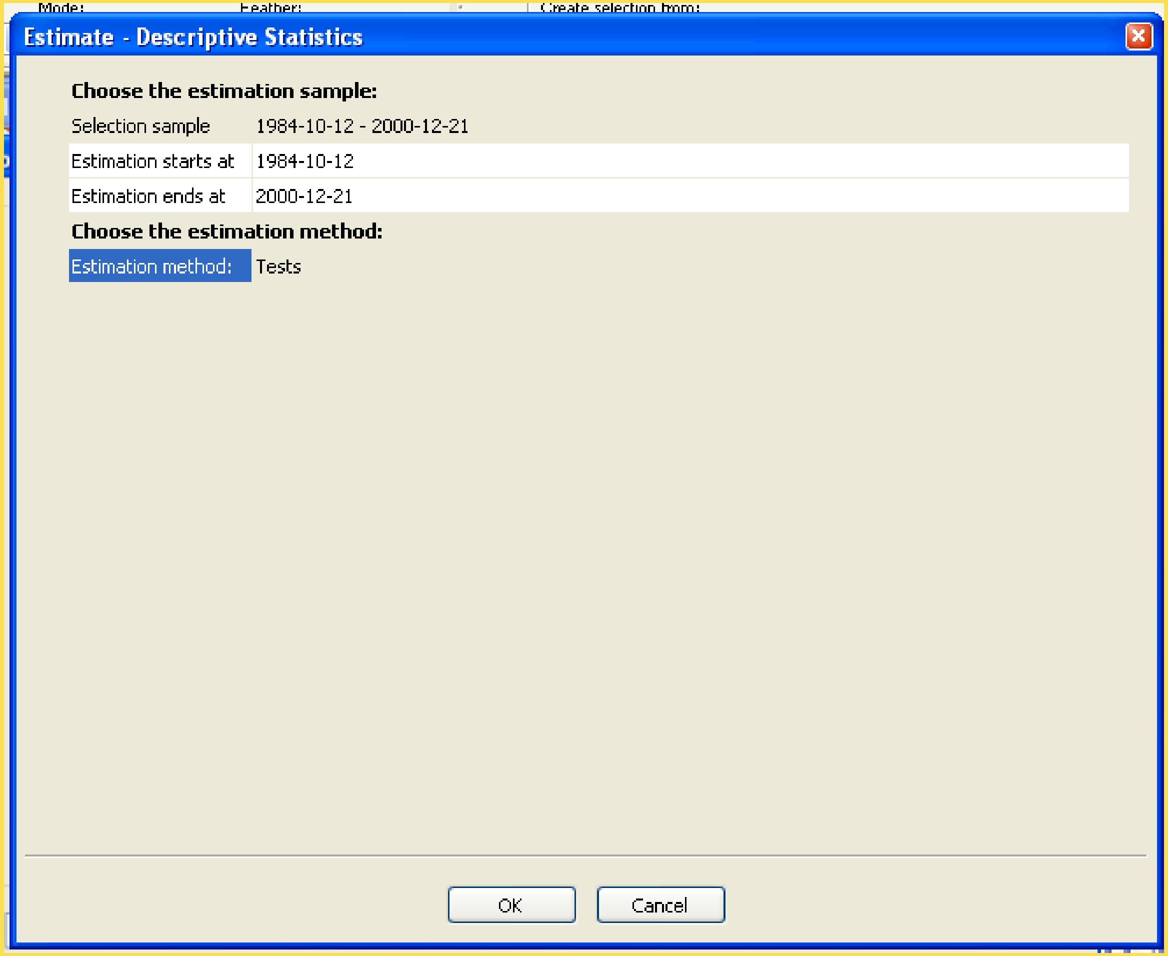


Select the variable

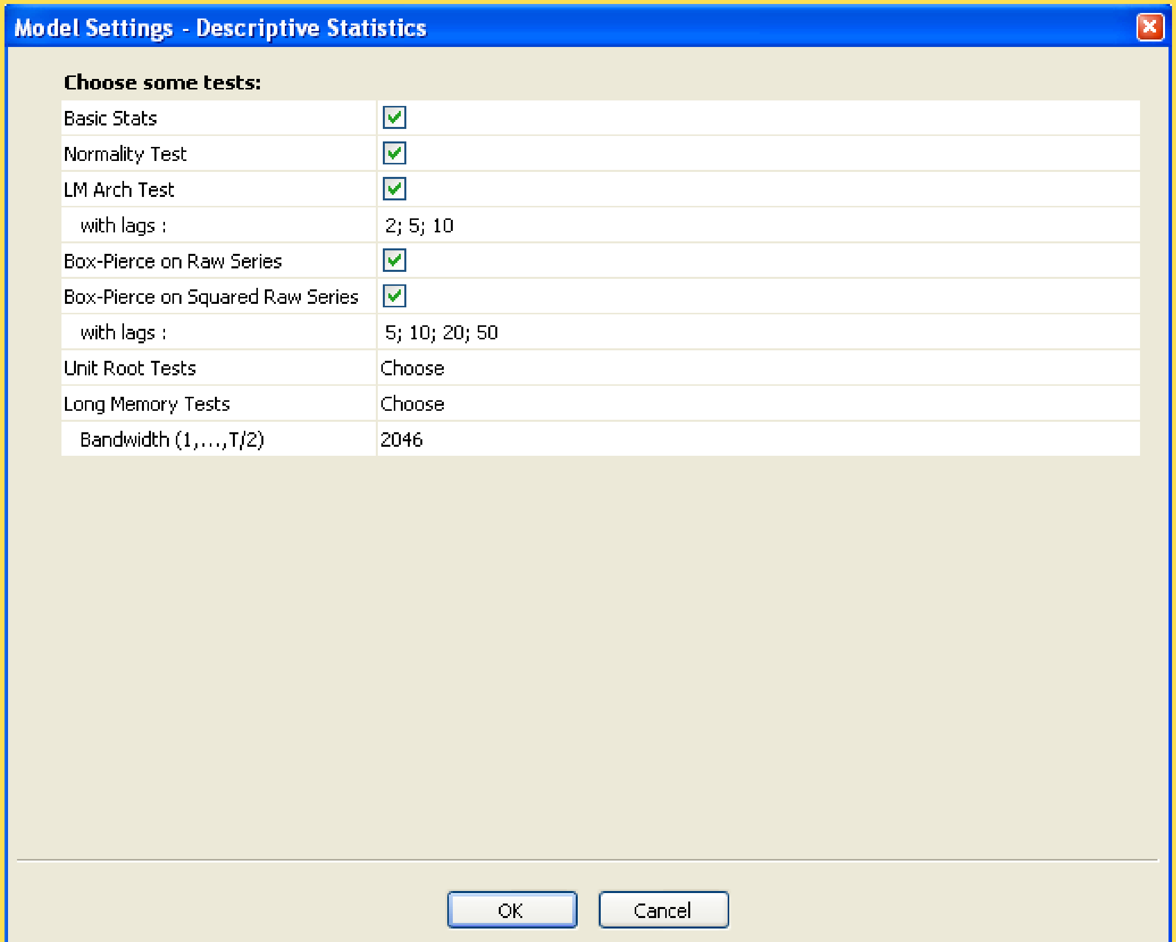


Predictor variables may be selected for mean or variance model.

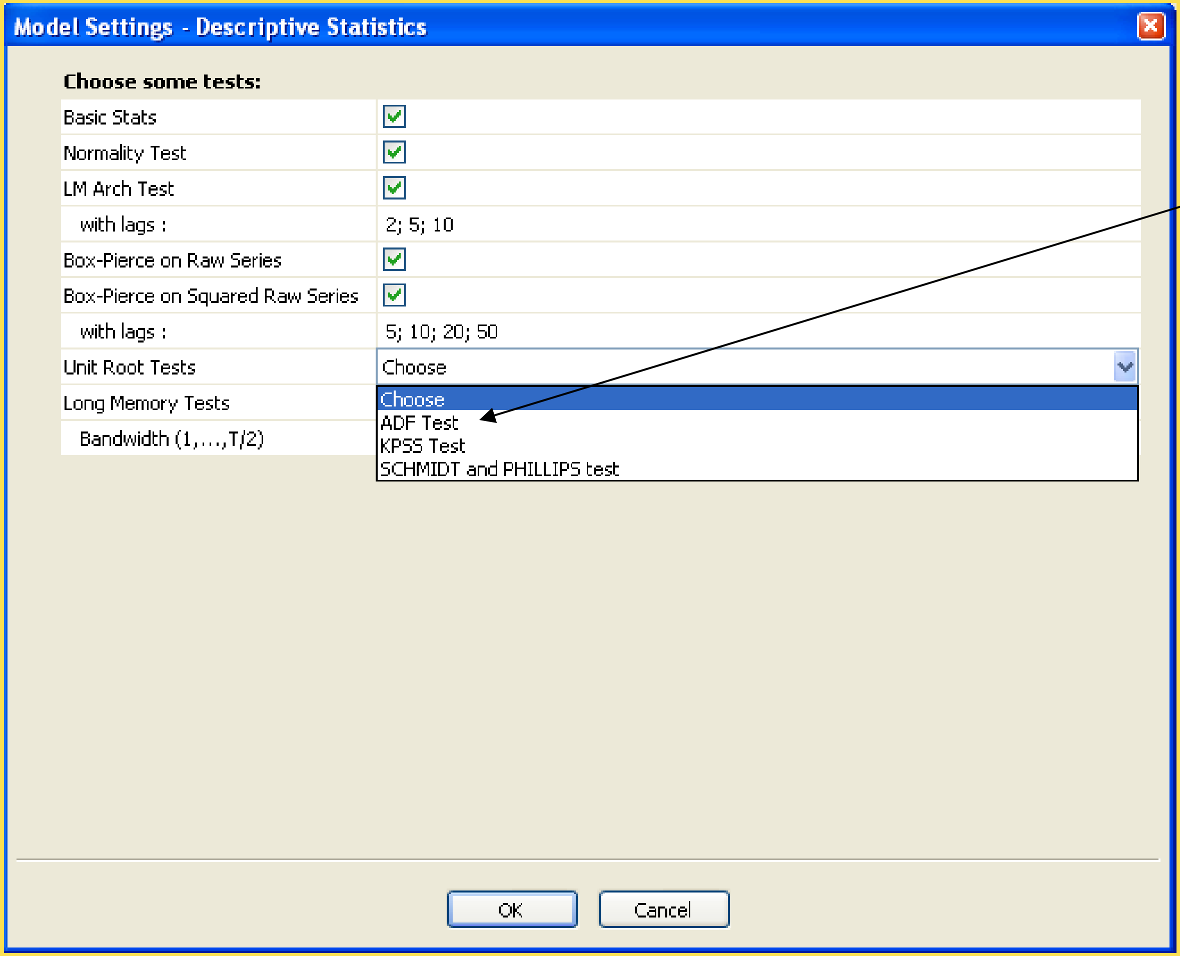
Define the sample



Specify the preliminary tests

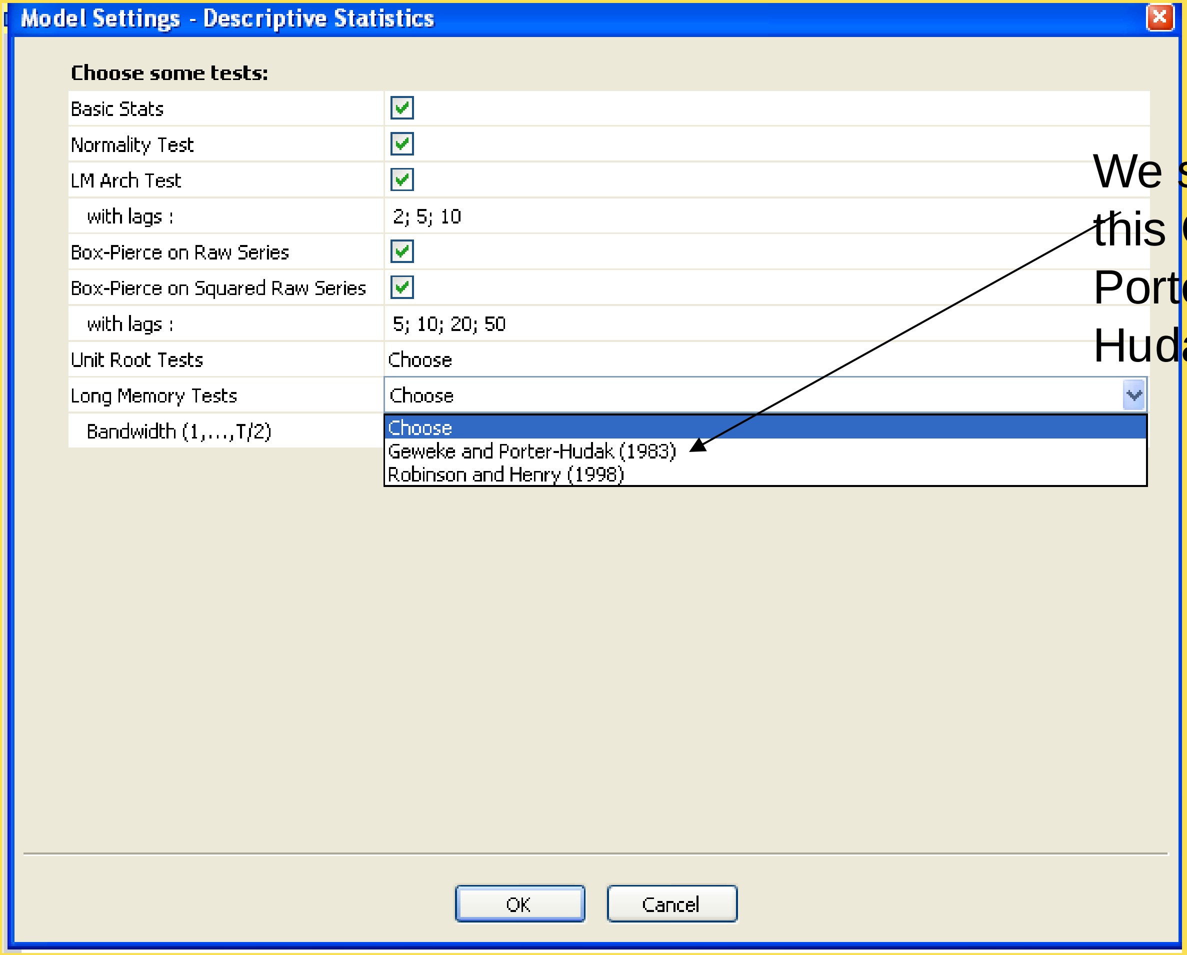


Choose the Stationarity tests



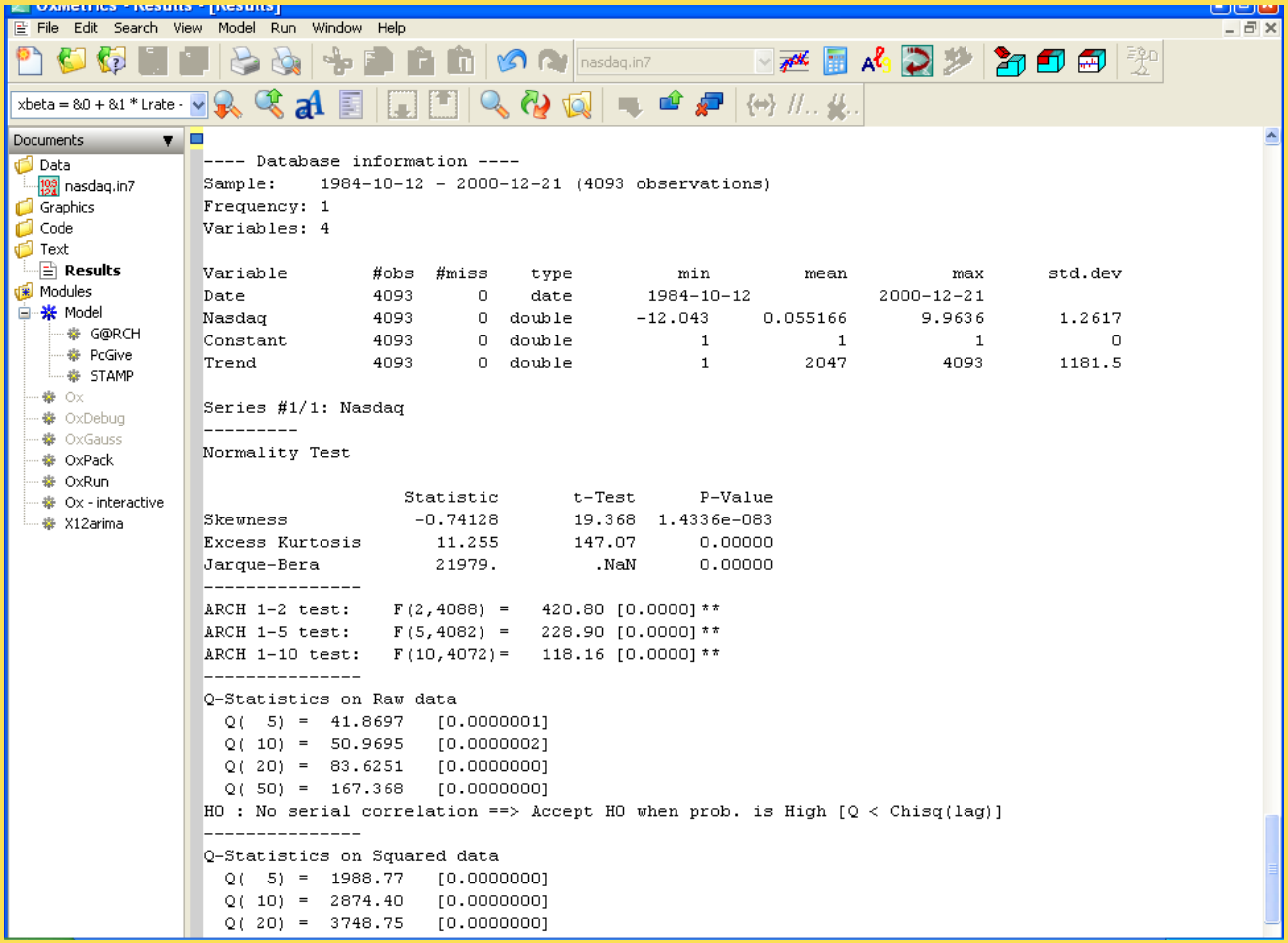
We select ADF test

Choose the Long-Memory Test

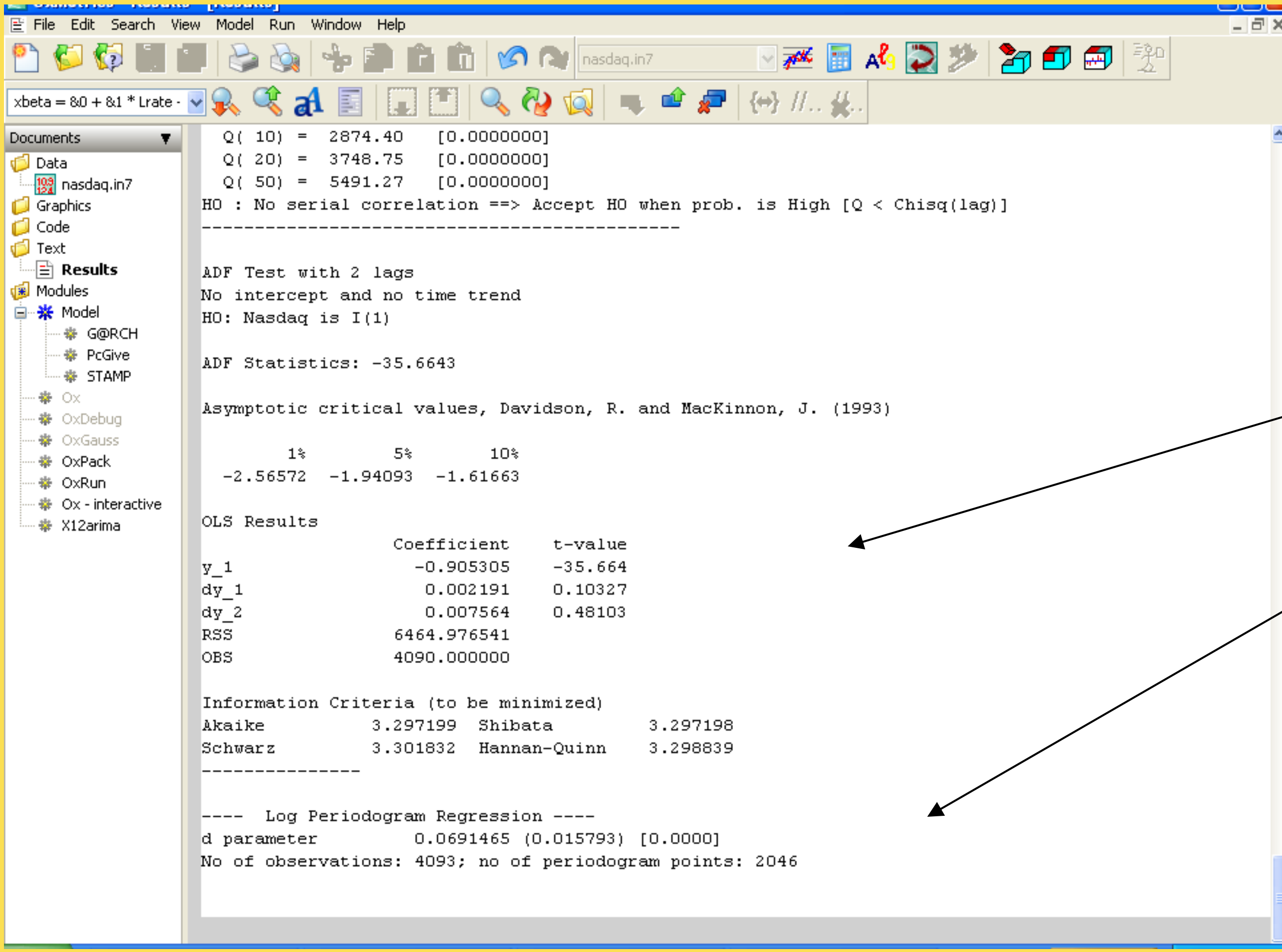


We select this Geweke Porter-Hudak test

Misspecification test results



Nonstationarity and Long-Memory Results



$$\Delta y_t = (\beta - 1) y_{t-1} + e_t$$

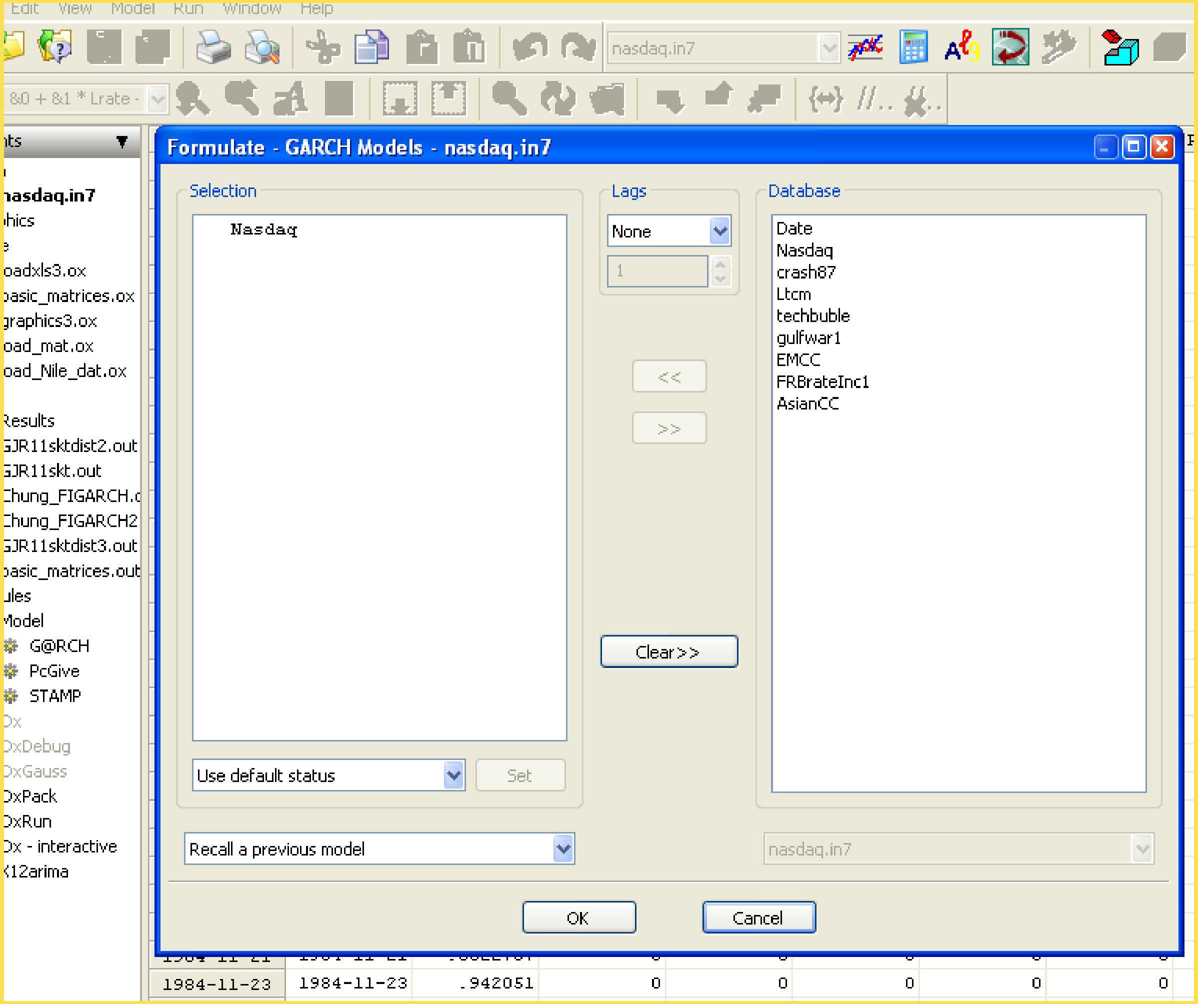
Nonstationary

Long memory parameter is weak—should be .between 0 and .5 for persistence. Above .5, d is not stationary.

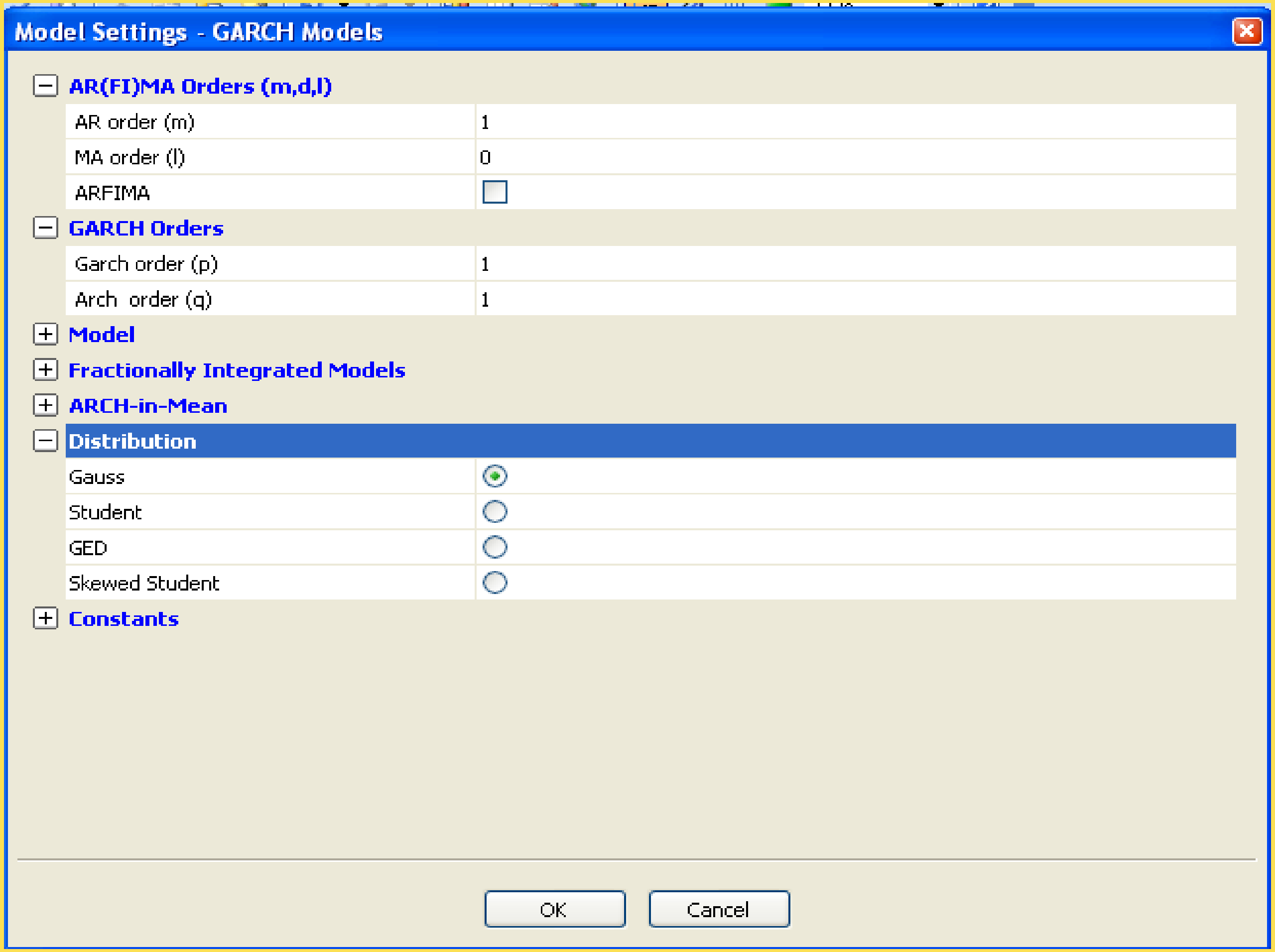
Pre-Model Analysis

- The Jarque-Bera tests suggests nonnormality--- we should probably try a t distribution
- The ARCH tests suggest ARCH effects
- The Portmanteau tests suggest autocorrelation
- The Nasdaq returns are nonstationary and there is long memory

Variable Selection



Baseline model parameter selection



AR(1) GARCH(1,1) normal distribution is our baseliine model

Normal GARCH(1,1)

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} a_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} b_i \sigma_{t-j}^2$$

Impact of previous shock

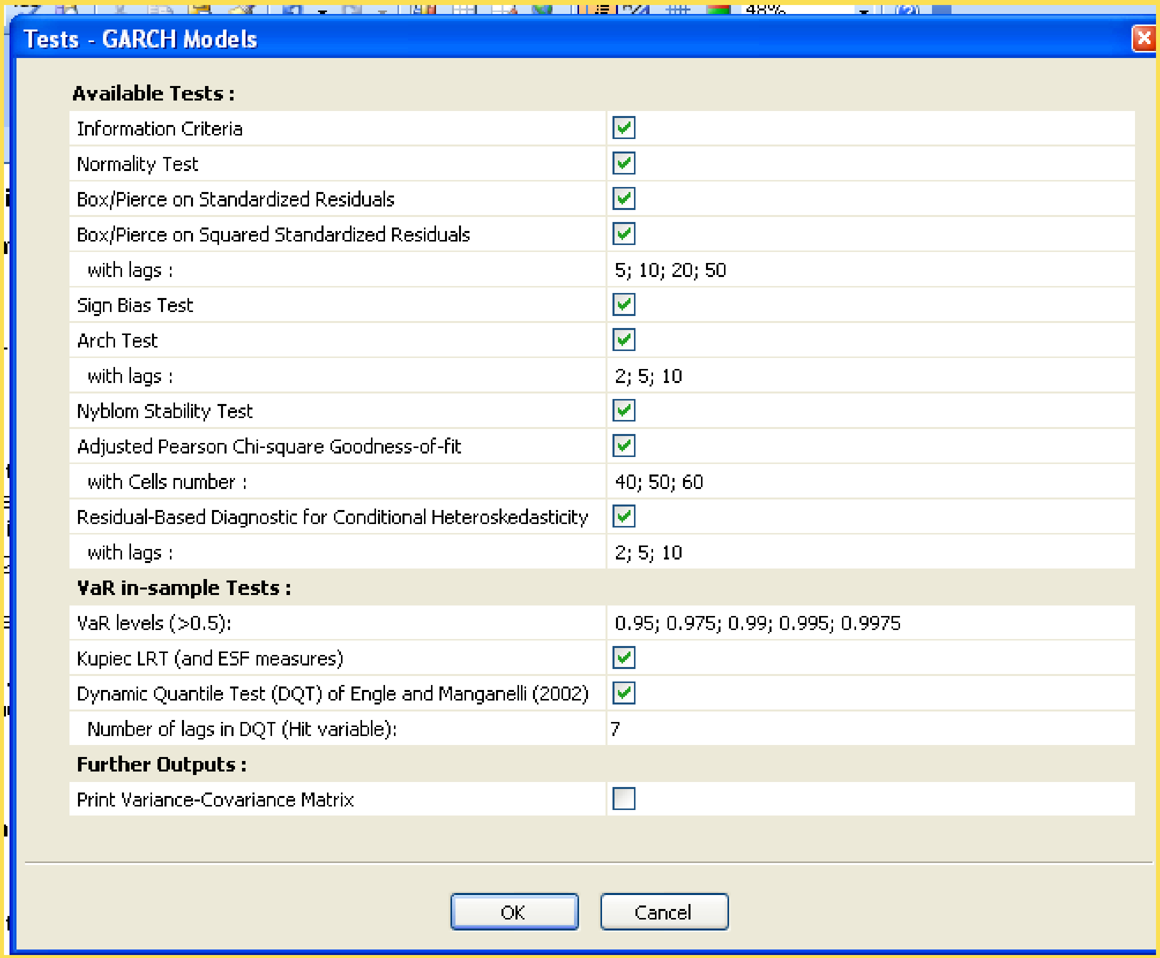
Persistence of conditional error variance

Normal GARCH(1,1) model output

```
 ******************************
 ** G@RCH( 1) SPECIFICATIONS **
 ******************************
Dependent variable : Nasdaq
Mean Equation : ARMA (1, 0) model.
No regressor in the conditional mean
Variance Equation : GARCH (1, 1) model.
No regressor in the conditional variance
Normal distribution.

Strong convergence using numerical derivatives
Log-likelihood = -5395.14
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.084113   0.015616    5.386  0.0000
AR(1)                0.193052   0.017187    11.23  0.0000
Cst(V)               0.025299  0.0071185    3.554  0.0004
ARCH(Alpha1)         0.167673   0.029686    5.648  0.0000
GARCH(Beta1)         0.820858   0.028236    29.07  0.0000

No. Observations :      4093  No. Parameters   :         5
Mean (Y)         :   0.05517  Variance (Y)     :   1.59189
Skewness (Y)     :  -0.74128  Kurtosis (Y)     :  14.25531
Log Likelihood   : -5395.144  Alpha[1]+Beta[1]:   0.98853

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is alpha[L]/[1 - beta(L)] >= 0.
The unconditional variance is 2.2058
The conditions are alpha[0] > 0, alpha[L] + beta[L] < 1 and alpha[i] + beta[i] >= 0.
  => See Doornik & Ooms (2001) for more details.
The condition for existence of the fourth moment of the GARCH is not observed.
The constraint equals 1.03342 and should be < 1.
  => See Ling & McAleer (2001) for details.
```

You may need more constraints or a different distribution

Selecting Post-Estimation tests



Test Results I

```
 ***********
 ** TESTS **
***********

TESTS :
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Information Criteria (to be minimized)
Akaike         2.638722  Shibata        2.638719
Schwarz        2.646439  Hannan-Quinn   2.641454
---------------

Normality Test

                   Statistic       t-Test      P-Value
Skewness            -0.69210       18.083   4.3269e-073
Excess Kurtosis       2.8275       36.947       0.00000
Jarque-Bera           1690.2         .NaN       0.00000
---------------

Q-Statistics on Standardized Residuals
  --> P-values adjusted by 1 degree(s) of freedom
  Q(  5) =   4.93617    [0.2939093]
  Q( 10) =   6.12809    [0.7270328]
  Q( 20) =   20.1258    [0.3870456]
  Q( 50) =   63.5812    [0.0787143]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Q-Statistics on Squared Standardized Residuals
  --> P-values adjusted by 2 degree(s) of freedom
  Q(  5) =   3.20892    [0.3605224]
  Q( 10) =   6.56843    [0.5838280]
  Q( 20) =   14.7703    [0.6776756]
  Q( 50) =   44.0824    [0.6340817]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------
```

Jarque Bera test

Box-Pierce Q tests on standardized residuals confirm proper modeling

Engle's ARCH LM test

Test Results II

```
Diagnostic test based on the news impact curve (EGARCH vs. GARCH)
                                      Test   P-value
Sign Bias t-Test                   2.48129   0.01309
Negative Size Bias t-Test          1.20393   0.22862
Positive Size Bias t-Test          1.42296   0.15475
Joint Test for the Three Effects 25.46329   0.00001
---------------
ARCH 1-2 test:    F(2,4086) =   1.2301 [0.2924]
ARCH 1-5 test:    F(5,4080) =  0.62728 [0.6790]
ARCH 1-10 test:   F(10,4070)=  0.63998 [0.7805]
---------------

Joint Statistic of the Nyblom test of stability: 6.34273

Individual Nyblom Statistics:
Cst(M)           0.09948
AR(1)            3.47893
Cst(V)           0.63865
ARCH(Alpha1)     1.25070
GARCH(Beta1)     1.58594

Rem: Asymptotic 1% critical value for individual statistics = 0.75.
     Asymptotic 5% critical value for individual statistics = 0.47.
---------------

Adjusted Pearson Chi-square Goodness-of-fit test

# Cells(g)   Statistic     P-Value(g-1)     P-Value(g-k-1)
    40        176.3672       0.000000          0.000000
    50        210.1884       0.000000          0.000000
    60        219.8317       0.000000          0.000000
Rem.: k = 5 = # estimated parameters
---------------
```

Sign Bias test for Leverage

Nyblom Hansen stability tests: Critical values are .75 for 1% and .47 for 5% levels. Null is parameter stability

Integrated effect

Theoretical v actual innovation Fit needs improvement

Test Results III

```
Residual-Based Diagnostic for Conditional Heteroskedasticity of Tse (2002)
  RBD( 2) = -6.40705    [1.0000000]
  RBD( 5) = 0.485674    [0.9926385]
  RBD(10) =  4.53051    [0.9202588]
-------------------------------------------------
P-values in brackets
```

Properly modeled ARCH effects

```
In-sample Value-at-Risk Backtesting

Kupiec LR test
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```

Basel II Kupiec Tests

- Short positions -

Quantile	Failure rate	Kupiec LRT	P-value	ESF1	ESF2
0.95000	0.96628	25.679	4.0322e-007	2.4334	1.2525
0.97500	0.98534	21.040	4.4975e-006	2.7129	1.2327
0.99000	0.99365	6.3191	0.011945	3.3298	1.2057
0.99500	0.99682	3.1457	0.076125	3.7132	1.2360
0.99750	0.99780	0.15517	0.69364	4.2753	1.2153

- Long positions -

Quantile	Failure rate	Kupiec LRT	P-value	ESF1	ESF2
0.050000	0.058881	6.4458	0.011121	-2.3226	1.4410
0.025000	0.035915	17.662	2.6386e-005	-2.5715	1.3855
0.010000	0.019301	28.117	1.1419e-007	-2.9948	1.3767
0.0050000	0.013926	44.033	3.2289e-011	-3.2095	1.3548
0.0025000	0.0092841	44.368	2.7216e-011	-3.6826	1.3830

Expected shortfall amounts

Kupiec test for frequency of tail losses

Kupiec Test

- Tests whether there is a significant difference between the failure rate and the nominal rate of failure.

$$H_0: \text{ } failure\ rate\ \ f = \alpha$$

$$Confidence\ level\ \ for\ \ f\ = \hat{f} \pm 1.96\sqrt{\hat{f}(1-\hat{f})/T}$$

$$T = total\ \ number\ \ of\ \ obs$$

Expected shortfall

- ES=conditional value at risk (CVAR)
- CVAR=expected (average) loss at or beyond the alpha-quantile or 1-alpha quantile.

Dynamic Quantile Regression

- Models the effect of the regressor on the alpha-th quantile of the regressand.
- The slope parameter is a function of the quantile.
- The slope parameter shows the effect of the predictor variable on the alpha-th quantile.

Engle, R. and Manganelli,S. (1999) CaViaR Conditional Autoregressive Value at Risk

$$\begin{aligned}
&\text{Dynamic Quantile test}\\
&Hit_t(\alpha) = I(y_t < VaR_t(\alpha)) - \alpha\\
&Hit_t(1-\alpha) = I(y_t > VaR_t(1-\alpha)) - \alpha\\
&Hit_t = X\delta + u_t \qquad u_t = \begin{cases} -\alpha & prob(1-\alpha) \\ (1-\alpha) & prob(\alpha) \end{cases}\\
&\text{such that } E(u_t) = 0,\\
&\text{where}\\
&Hit_t(y_t, x_t, \theta) \equiv Hit_{\alpha t} \equiv (y_t < -VaR_t) - \alpha\\
&\text{where}\\
&X = Txk \text{ matrix whose first column is col of ones}\\
&\text{and next are } Hit_{t-1}, \ldots, Hit_{t-p}
\end{aligned}$$

The Dynamic Quantile Test Statistic

$$Because\ \hat{\delta}_{ols} = (X'X)^{-1}X'Hit\ :\ N(0,\alpha(1-\alpha)(X'X)^{-1}),$$

$$Dynamic\ Quantile\ test\ Statistic =$$

$$\frac{\hat{\delta}_{ols}X'X\hat{\delta}_{ols}}{\alpha(1-\alpha)}\ :\ \chi^2(p+n+2)$$

Remember X may = t=1,…,T

Assumption: Hits are uncorrelated and unbiased

Dynamic Quantile Hypothesis Tests

A joint test that

$$A1: E(Hit_t(\alpha)) = 0 \ \ for \ \ trading \ long \ positions$$

$$E(Hit_t(1-\alpha)) = 0 \ \ for \ \ trading \ \ short \ positions$$

$$A2: Hit_t(\alpha) \ or \ Hit_t(1-\alpha) \ is \ uncorrelated \ \ with \ variables$$

in the information set

Applications of the DQ test
Engle and Mangenelli, CaViaR, p.28

- "A model diagnostic or preliminary screening device to distinguish between good and bad models.
- An evaluation of the performance of different VaR methodologies.
- If test is significant, then data provide evidence against the model produced under those estimates.
- If DQ test falls into rejection reject for an out-of-sample test, this is evidence against the model and its stability over time."

Test Results IV

Quantile Regression VAR

Dynamic Quantile Test of Engle and Manganelli (2002)

- Short positions -

Quantile	Stat.	P-value
0.95000	25.551	0.0012532
0.97500	20.863	0.0075214
0.99000	8.8216	0.35757
0.99500	2.9260	0.93892
0.99750	0.27316	0.99999

- Long positions -

Quantile	Stat.	P-value
0.050000	13.109	0.10814
0.025000	36.600	1.3629e-005
0.010000	48.500	7.9279e-008
0.0050000	77.972	1.2501e-013
0.0025000	90.662	3.3307e-016

Remark: In the Dynamic Quantile Regression, p=7.

Unconditional Variance

- Of a GARCH model

$$\sigma^2 = \frac{\omega}{1 - \sum_{i=1}^{q} a_i - \sum_{j=1}^{p} \beta_j}$$

$$unconditional\ variance\ of\ \varepsilon_t\ is\ constant$$

$$if\ \ \omega > 1\ \&\ \sum_{i=1}^{q} a_i + \sum_{j=1}^{p} \beta_j < 1$$

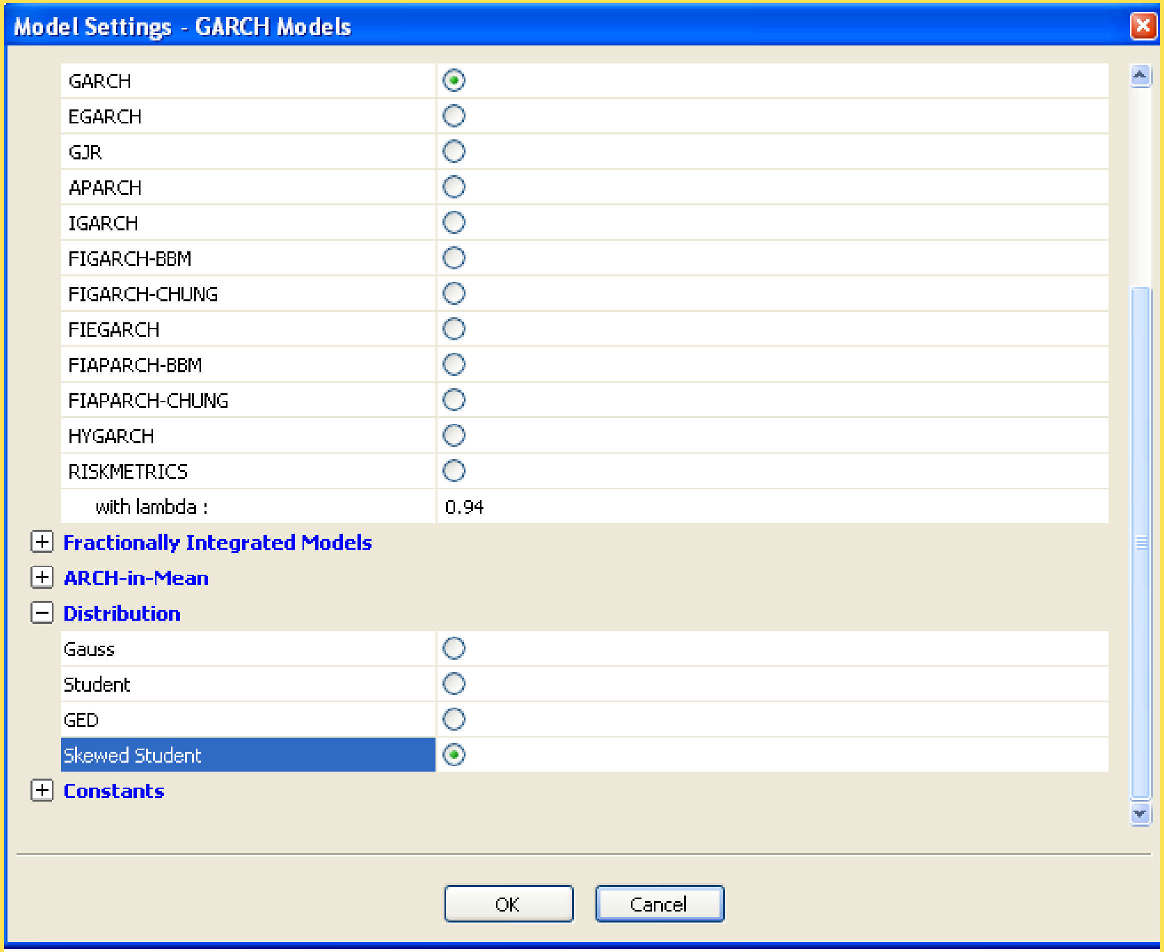
AR(1) GARCH(1,1) sk(t)

$$y_t = \mu + \phi y_{t-1} + \varepsilon_t$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} a_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} b_i \sigma_{t-j}^2$$

Distribution is a skewed t distribution

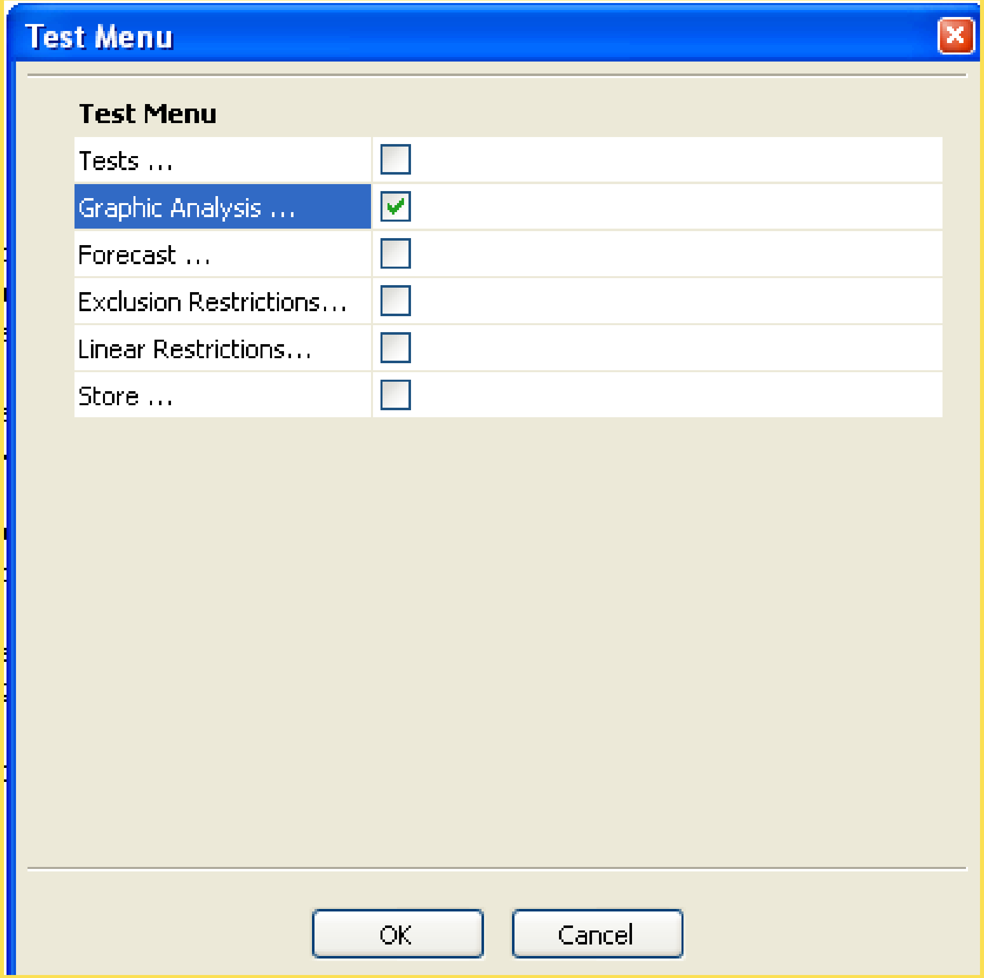
AR(1) GARCH(1,1) sk(t)



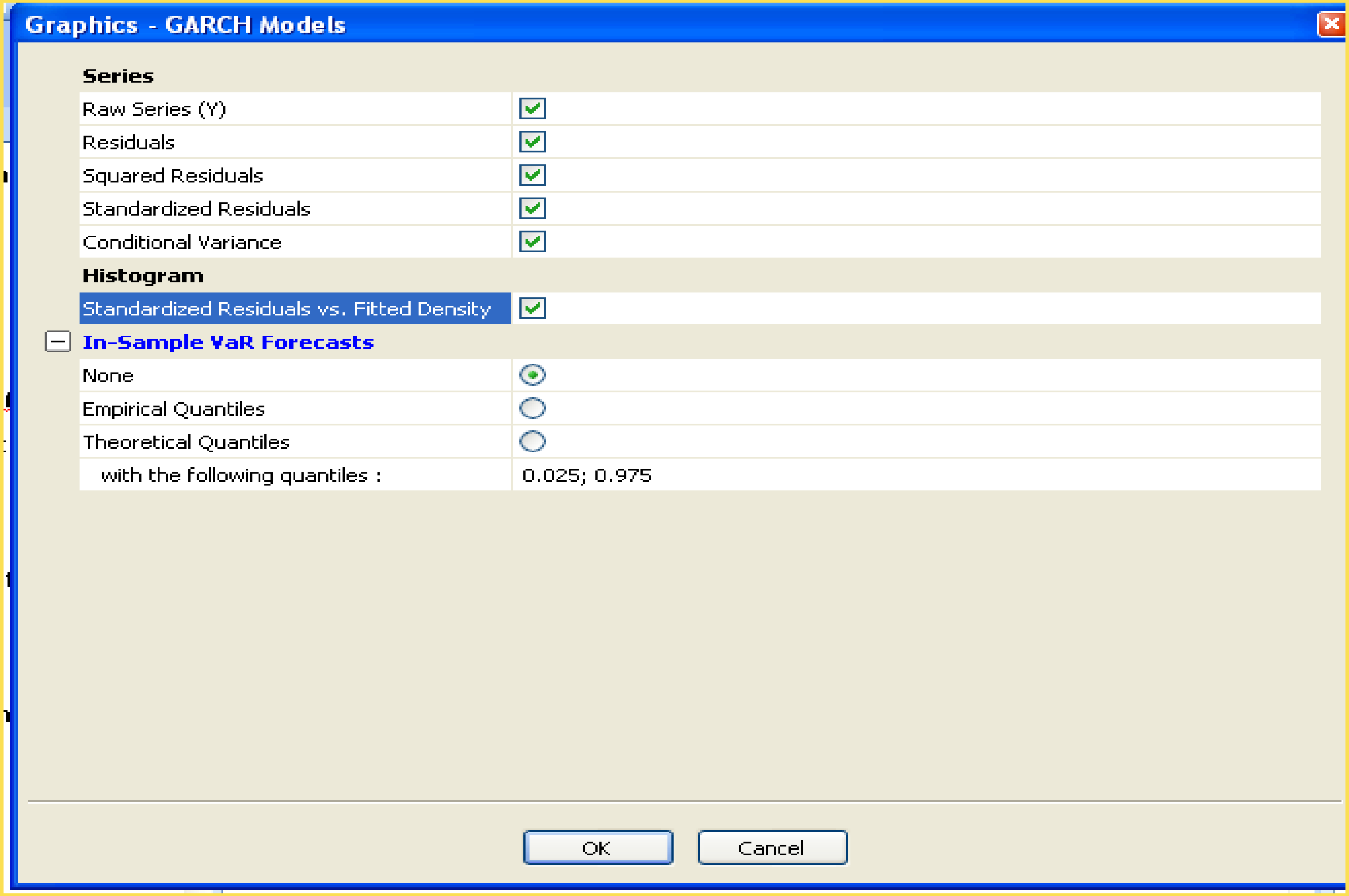
AR(1)-GARCH(1,1) sk(t) output

```
 *****************************
 ** G@RCH( 2) SPECIFICATIONS **
 *****************************
Dependent variable : Nasdaq
Mean Equation : ARMA (1, 0) model.
No regressor in the conditional mean
Variance Equation : GARCH (1, 1) model.
No regressor in the conditional variance
Skewed Student distribution, with 6.36561 degrees of freedom.
and asymmetry coefficient (log xi) -0.176807.

Strong convergence using numerical derivatives
Log-likelihood = -5228.77
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.074547   0.013749    5.422  0.0000
AR(1)                0.172029   0.015928    10.80  0.0000
Cst(V)               0.013518  0.0041361    3.268  0.0011
ARCH(Alpha1)         0.135317   0.022145    6.111  0.0000
GARCH(Beta1)         0.862093   0.021804    39.54  0.0000
Asymmetry           -0.176807   0.022959   -7.701  0.0000
Tail                 6.365606    0.62455    10.19  0.0000

No. Observations :      4093  No. Parameters   :         7
Mean (Y)         :   0.05517  Variance (Y)     :   1.59189
Skewness (Y)     :  -0.74128  Kurtosis (Y)     :  14.25531
Log Likelihood   : -5228.772  Alpha[1]+Beta[1]:   0.99741

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is alpha[L]/[1 - beta(L)] >= 0.
The unconditional variance is 5.21841
The conditions are alpha[0] > 0, alpha[L] + beta[L] < 1 and alpha[i] + beta[i] >= 0.
  => See Doornik & Ooms (2001) for more details.
```

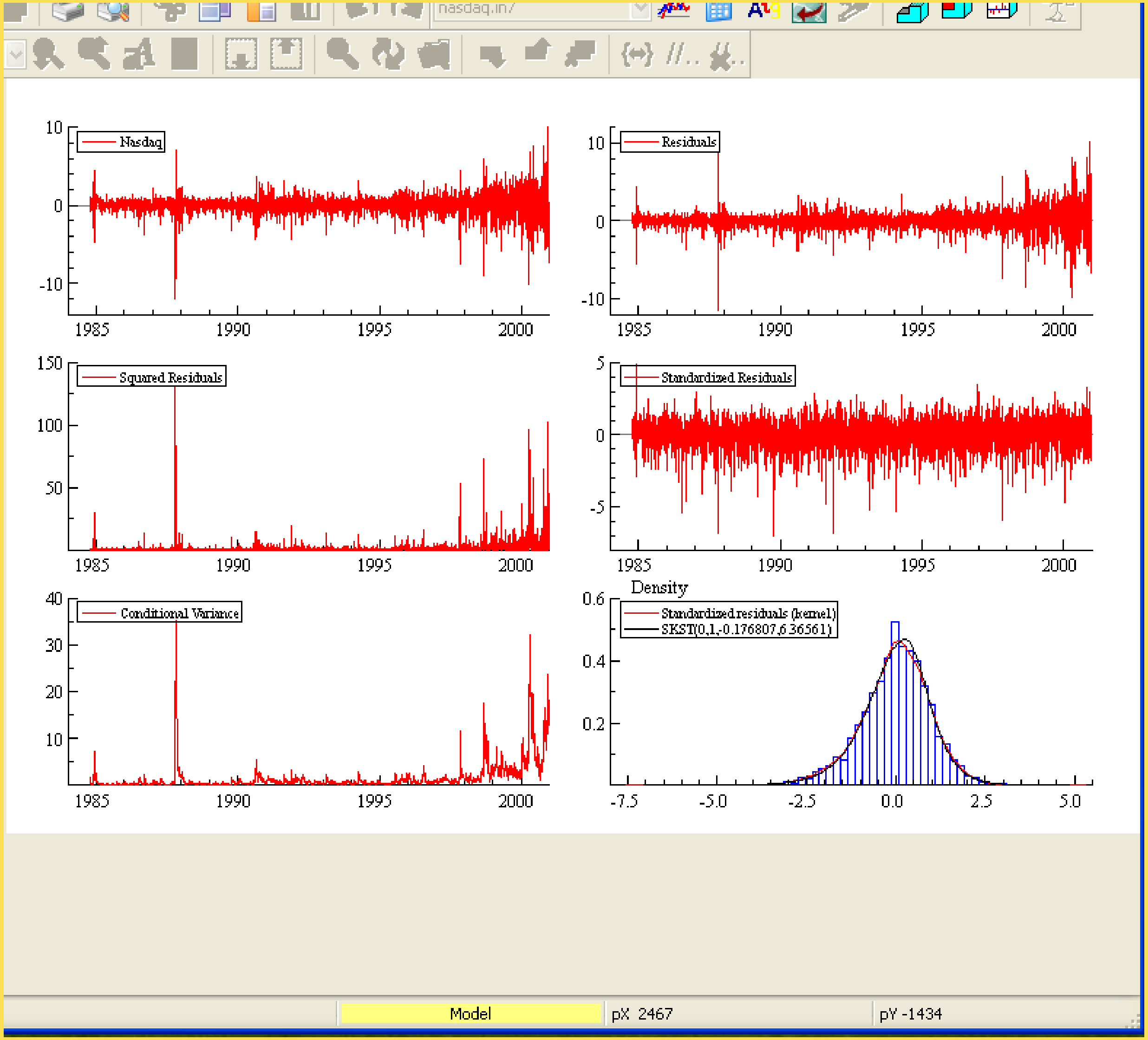
Graphical Analysis



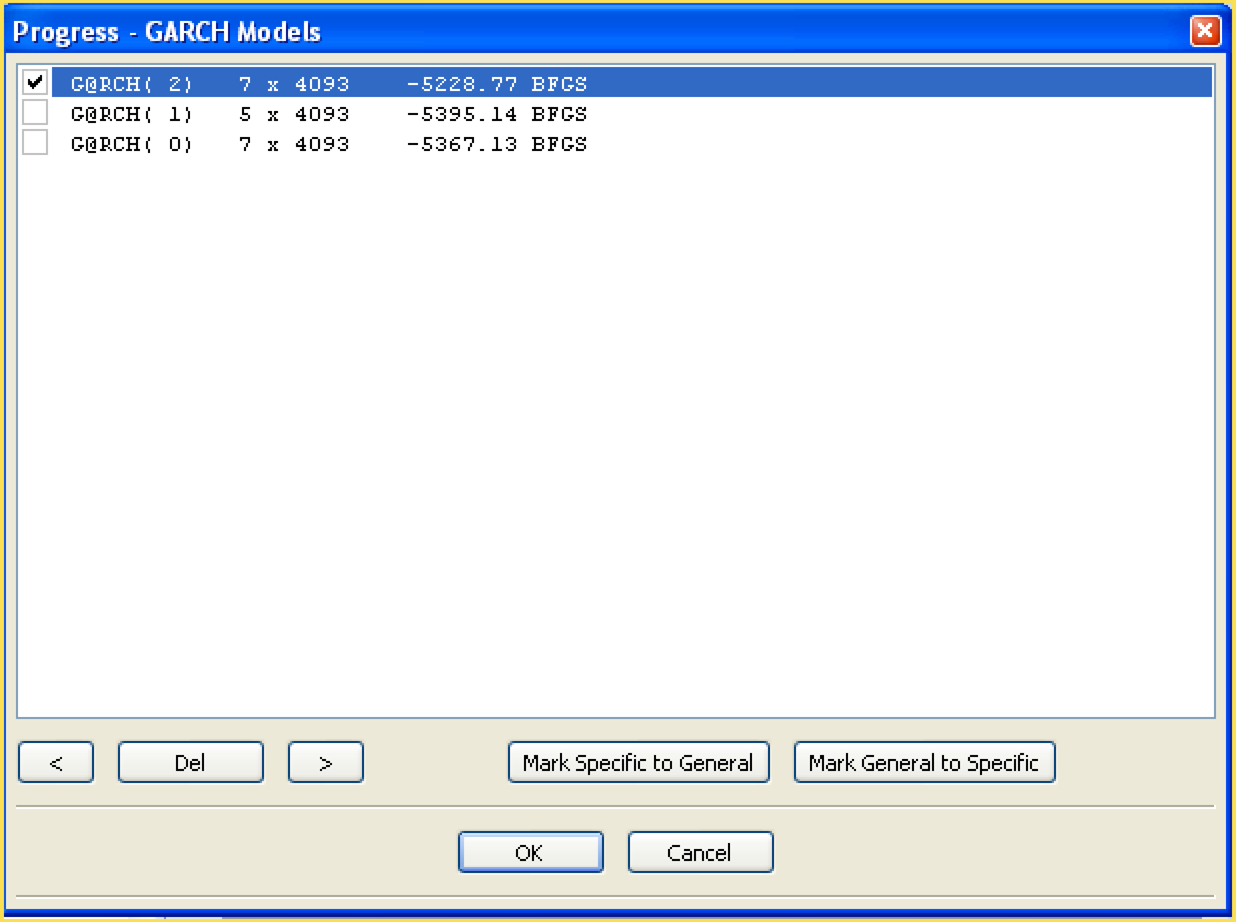
Graph selection



Graphical Output



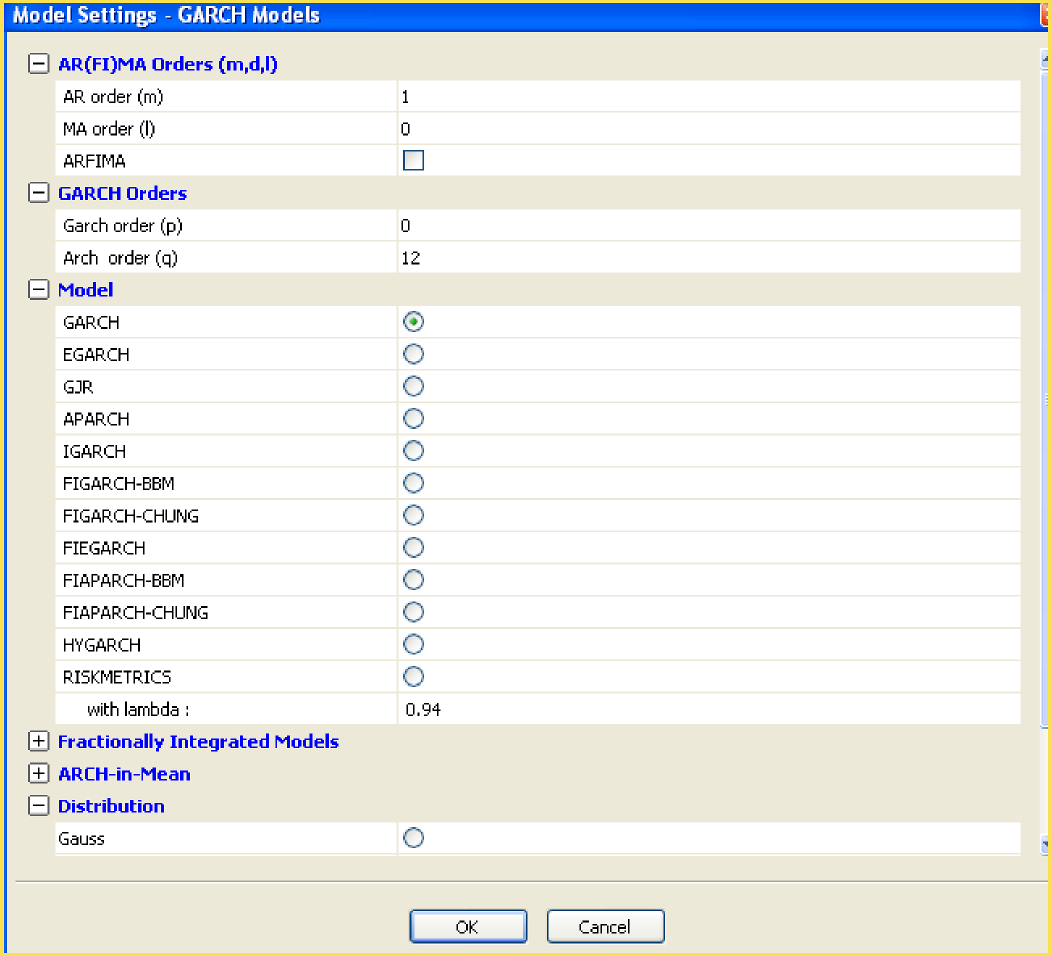
Model Comparison



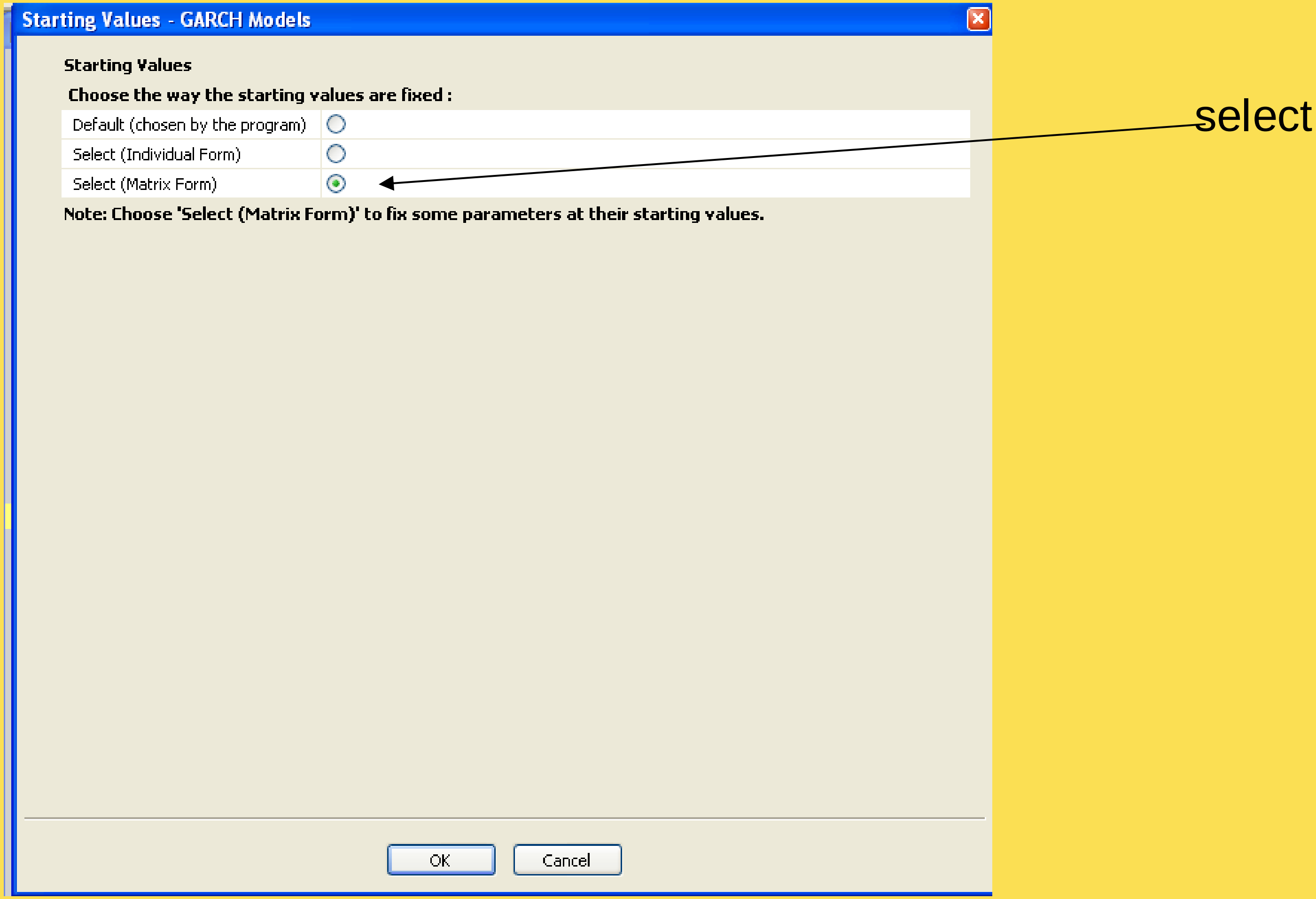
Subset Models

- Constraining parameters to be zero.
- We perform an ARCH(12)-t on NQ.
- We find that the a_{t-8}=0
- We wish to eliminate that from the model, so we constrain it to be zero.

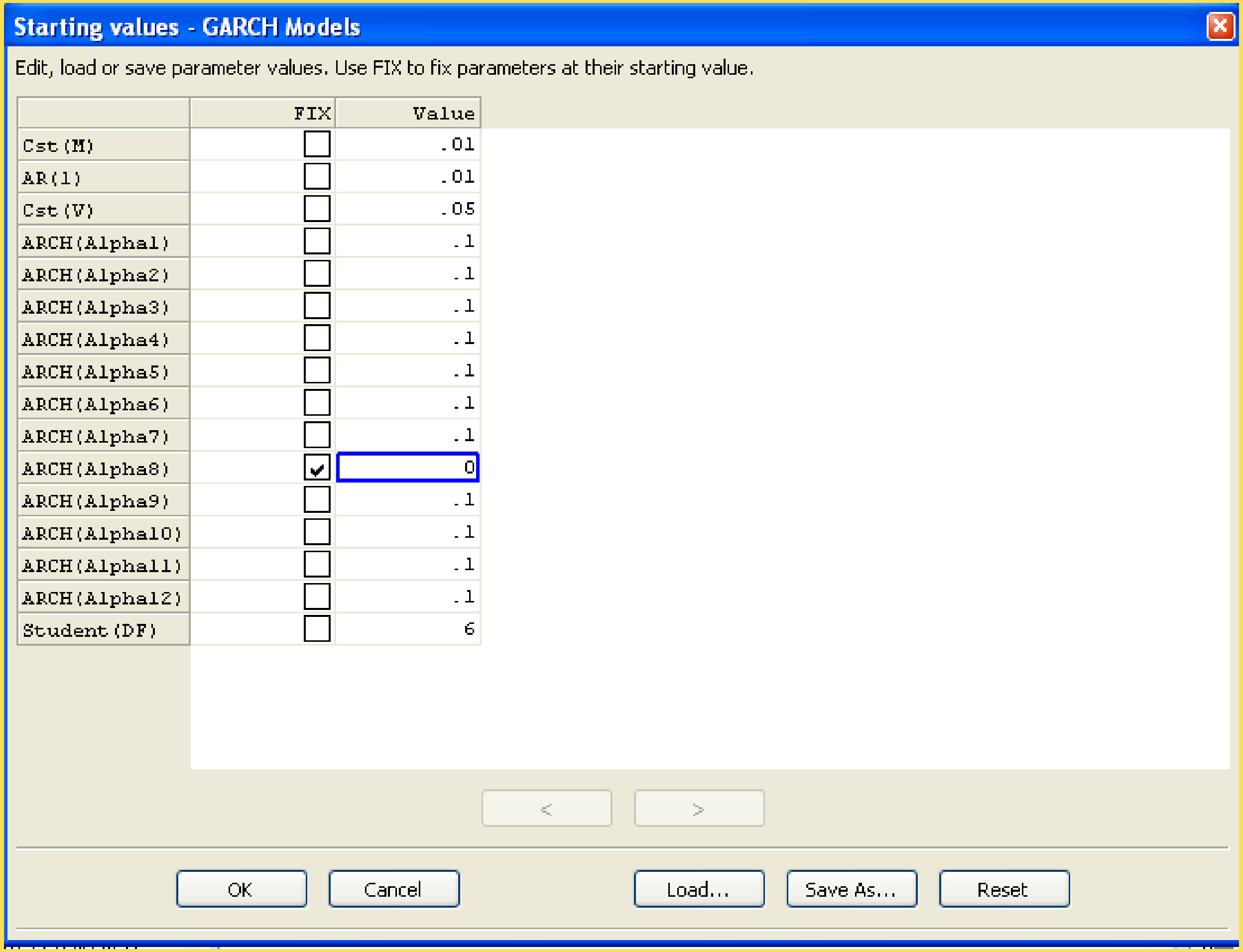
We set up an AR(1) ARCH(12) t model



We opt for Matrix form starting values– this lets us fix values



We set ARCH(8)=0



The Subset Model does not contain ARCH(8)

Results

```
Dependent variable : NQ
Mean Equation : ARMA (1, 0) model.
No regressor in the conditional mean
Variance Equation : GARCH (0, 12) model.
No regressor in the conditional variance
Student distribution, with 6.92557 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = -6204.49
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.098077   0.016223    6.046  0.0000
AR(1)                0.097995   0.015528    6.311  0.0000
Cst(V)               0.226498   0.028271    8.012  0.0000
ARCH(Alpha1)         0.067690   0.020706    3.269  0.0011
ARCH(Alpha2)         0.181228   0.026585    6.817  0.0000
ARCH(Alpha3)         0.086573   0.023254    3.723  0.0002
ARCH(Alpha4)         0.121771   0.023721    5.133  0.0000
ARCH(Alpha5)         0.147807   0.027587    5.358  0.0000
ARCH(Alpha6)         0.096119   0.023526    4.086  0.0000
ARCH(Alpha7)         0.058725   0.022167    2.649  0.0081
ARCH(Alpha9)         0.045479   0.020760    2.191  0.0285
ARCH(Alpha10)        0.046155   0.026043    1.772  0.0764
ARCH(Alpha11)        0.049121   0.020960    2.343  0.0192
ARCH(Alpha12)        0.066955   0.023148    2.892  0.0038
Student(DF)          6.925567    0.79046    8.761  0.0000
ARCH(Alpha8)         0.000000

No. Observations :      3901  No. Parameters  :        15
Mean (Y)         :   0.03552  Variance (Y)    :   2.38613
Skewness (Y)     :  -0.01241  Kurtosis (Y)    :   8.75357
Log Likelihood   : -6204.486  Alpha[1]+Beta[1]:   0.96762
```

Second generation GARCH

- Nonstationary GARCH
- Garch-in-mean
- Asymmetric GARCH
  - Leverage effects captured in EGARCH
  - GJR GARCH
    APARCH,APGARCH
- Skewed t distribution captures leverage effects better

Nonstationary GARCH

- Riskmetrics

$$Riskmetrics: \sigma_t^2 = \omega + (1-\lambda)\varepsilon_{t-1}^2 + \lambda\sigma_{t-1}^2$$

$$where\ \lambda = .94\ for\ daily\ data\ and\ .97\ for\ monthly\ data.$$

- IGARCH

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} a_i \varepsilon_{t-1}^2 + \sum_{j=1}^{p} b_i \sigma_{t-1}^2$$

where

$$\sum_{i=1}^{q} a_i \varepsilon_{t-1}^2 + \sum_{j=1}^{p} b_i \sigma_{t-1}^2 \approx 1$$

Usually set at .97

Garch-in-Mean

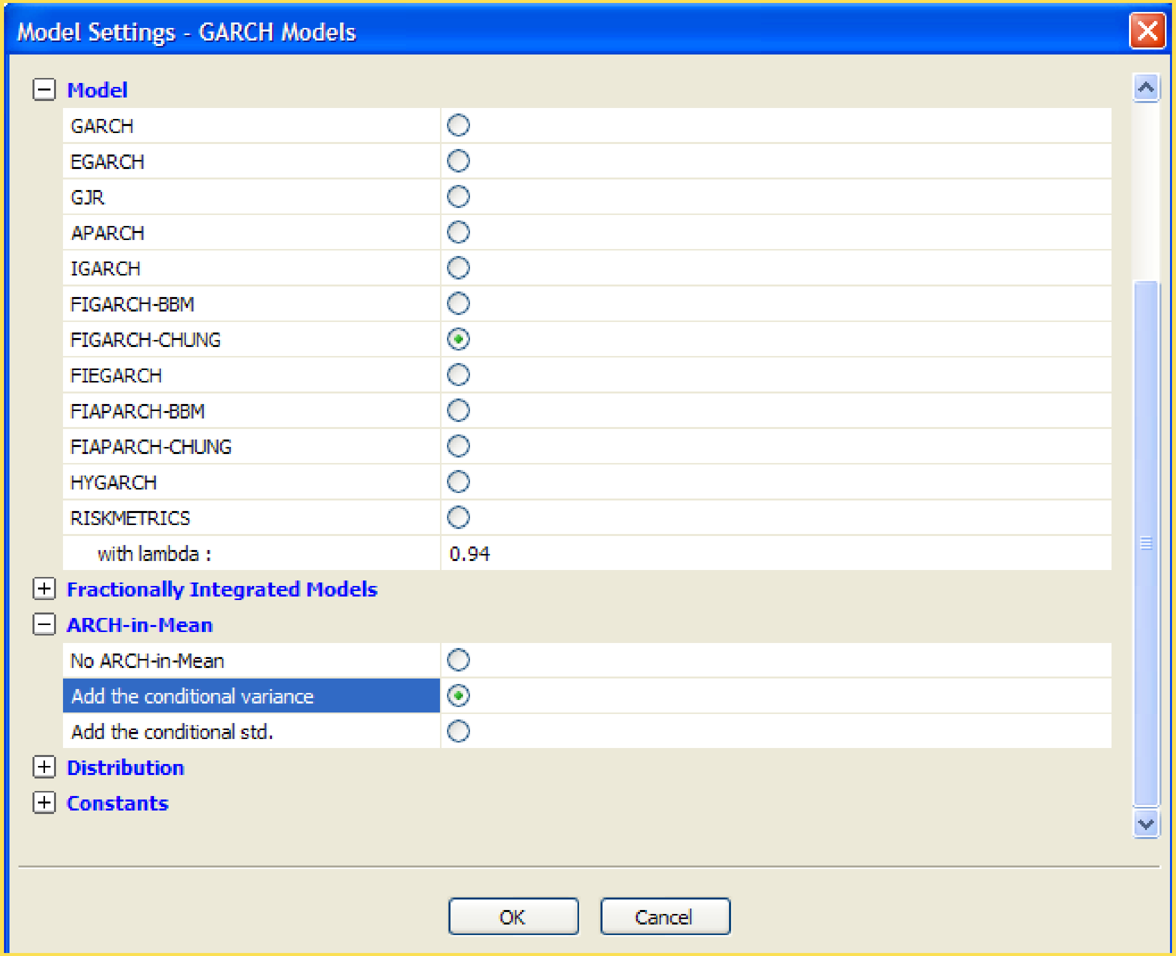
- One can add the conditional variance or the conditional standard deviation to the mean equation.

$$y_t = a + b_t x_t + \delta\sigma_t^2 + \varepsilon_t$$

where

$$\sigma_t^2 = \omega + a\sum_{i=1}^{q}\varepsilon_{t-i}^2 + b\sum_{j=1}^{p}\sigma_{t-p}^2$$

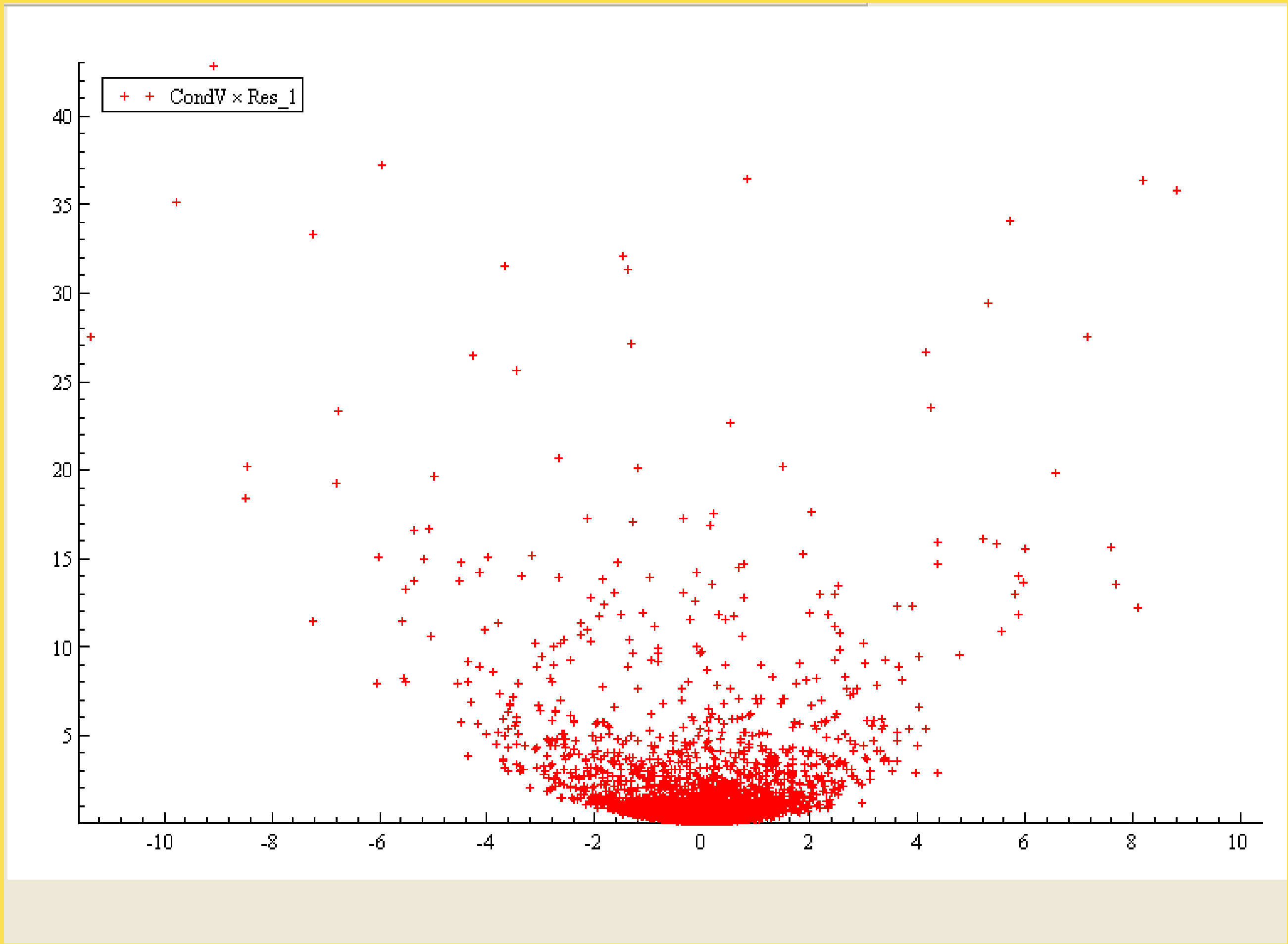
Select conditional variance



Testing leverage effects

- EGARCH Exponential GARCH (Nelson)
- GJR Threshold GARCH (Glosten, Jagannathan, Runkle)
- APARCH Asymmetric Power GARCH (Ding, Engle, and Granger)

Volatility Smile



Engle and Ng Asymmetry tests

$$Sign\ bias\ test: \hat{\varepsilon}_t^2 = a_0 + a_1 S_{t-1}^- + u_t$$

$$-Sign\ bias\ test: \hat{\varepsilon}_t^2 = b_0 + b_1 S_{t-1}^- \hat{\varepsilon}_{t-1} + u_t$$

$$+Sign\ bias\ test: \hat{\varepsilon}_t^2 = c_0 + c_1 S_{t-1}^+ \hat{\varepsilon}_{t-1} + u_t$$

$$Joint\ Sign\ bias\ test: \hat{\varepsilon}_t^2 = d_0 + d_1 S_{t-1}^- + d_2 S_{t-1}^- \hat{\varepsilon}_{t-1} + \ S_{t-1}^+ \hat{\varepsilon}_{t-1} + u_t$$

Asymmetry tests

```
TESTS :
---------

Diagnostic test based on the news impact curve (EGARCH vs. GARCH)
                                              Test    P-value
Sign Bias t-Test                           3.04899   0.00230
Negative Size Bias t-Test                  0.06642   0.94704
Positive Size Bias t-Test                  0.09179   0.92687
Joint Test for the Three Effects          15.39502   0.00151
---------------
```

Exponential GARCH (David Nelson, 1991)

$$\ln(h_t) = \omega + \sum_{i=1}^{q} \alpha_i g(z_{t-i}) + \sum_{j=1}^{p} \beta \ln(h_{t-j})$$

where

$$g(z_{t-i}) = \theta_1 z_t + \theta_2 \left(| z_t | - | \sqrt{\frac{2}{\pi}} | \right)$$

and

$$z_t = \frac{\varepsilon_t}{\sqrt{h_t}}$$

Sign effect

Magnitude effect

Glosten, Jagannathan, and Runkle (1993) (GJR) GARCH

$$\sigma^2 = \omega + \sum_{i=1}^{q} (\alpha_i \varepsilon_{t-1}^2 + \gamma_t S_{t-i}^- \varepsilon_{t-i}^2) + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

Leverage effect

GJR Asymmetric GARCH(1,1)

```
 ** G@RCH( 3) SPECIFICATIONS **
 ******************************
Dependent variable : Nasdaq
Mean Equation : ARMA (1, 0) model.
No regressor in the conditional mean
Variance Equation : GJR (1, 1) model.
No regressor in the conditional variance
Skewed Student distribution, with 6.54157 degrees of freedom.
and asymmetry coefficient (log xi) -0.17954.

Strong convergence using numerical derivatives
Log-likelihood = -5184.52
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.062393   0.014353    4.347  0.0000
AR(1)                0.185970   0.016883    11.02  0.0000
Cst(V)               0.017264  0.0052401    3.295  0.0010
ARCH(Alpha1)         0.096650   0.015176    6.369  0.0000
GARCH(Beta1)         0.850005   0.023975    35.45  0.0000
GJR(Gamma1)          0.092444   0.030406    3.040  0.0024
Asymmetry           -0.179540   0.023111   -7.769  0.0000
Tail                 6.541571    0.66287    9.869  0.0000

No. Observations :      4081  No. Parameters  :         8
Mean (Y)         :   0.06050  Variance (Y)    :   1.55316
Skewness (Y)     :  -0.72562  Kurtosis (Y)    :  14.49309
Log Likelihood   : -5184.518

The sample mean of squared residuals was used to start recursion.
The condition for existence of the second moment of the GJR is not observed.
This condition is alpha(1) + beta(1) + k gamma(1) < 1 (with k = 0.588818 with this distributio
In this estimation, this sum equals 1.00109.
The condition for existence of the fourth moment of the GJR is not observed.
The constraint equals 1.12247 (should be < 1).  => See Ling & McAleer (2001) for details.
```

Asymmetric Power GARCH
Ding, Granger, and Engle,1993

$$\sigma_t^{\delta} = \omega + \alpha_1 (|\varepsilon_{t-1}| - \gamma_1 \varepsilon_{t-1})^{\delta} + \beta_1 \sigma_{t-1}^{\delta}$$

where

$\delta = power\ captures\ long-memory\ effects\ when$

$\delta \approx 1$

Leverage effect

Model Comparison

```
Progress to date
Model              T    p           log-likelihood          SC          HQ         AIC
G@RCH( 0)       4093    7   BFGS        -5367.1337      2.6368      2.6298      2.6260
G@RCH( 1)       4093    5   BFGS        -5395.1442      2.6464      2.6415      2.6387
G@RCH( 2)       4093    7   BFGS        -5228.7723      2.5692      2.5622      2.5584
G@RCH( 3)       4081    8   BFGS        -5184.5175      2.5571<     2.5491<     2.5447<

Tests of model reduction (please ensure models are nested for test validity)
G@RCH( 0) --> G@RCH( 1): Chi^2( 2) =      56.021 [0.0000] **
```

By clicking on the progress button on the GARCH GUI, one can obtain the information criteria for preceding models to compare them for fit.

Forecasts

- Conditional mean, with confidence intervals
- Conditional variance
  - Intervals can be simulated
- VaR intervals serve as confidence intervals

GARCH Forecasting

- In-sample: This is estimation.
  - These can be evaluated by forecast error measures.
- Out-of-sample: This sets aside a hold-out-sample, over which forecasts are generated.
  - These can be evaluated by forecast error measures.
- Ex Ante: This generates forecasts beyond the end of the sample.
  - These cannot be evaluated until the real or comparative data are collected against which they can be measured.

Types of GARCH forecasts

- The conditional mean
- The conditional error variance
- The Value-at-Risk

Forecasting the conditional mean

$$Suppose\ we\ have\ an\ AR(1)\ mean\ process:$$

$$y_t = \mu + \phi(y_{t-1} - \mu) + \varepsilon_t$$

$$An\ optimal\ h\ step-ahead\ forecast\ is$$

$$\hat{y}_{t+h} = \hat{\mu} + \phi(\hat{y}_{t+h-1|t} - \mu)$$

$$= \hat{\mu} + \phi_1^h(\hat{y}_{t+h-1|t} - \mu)$$

$$For\ an\ ARMA(1,1)\ mean\ process:$$

$$\hat{y}_{t+h} = \hat{\mu} + \phi(\hat{y}_{t+h-1|t} - \mu) + \theta_1\varepsilon_{t+h-1}$$

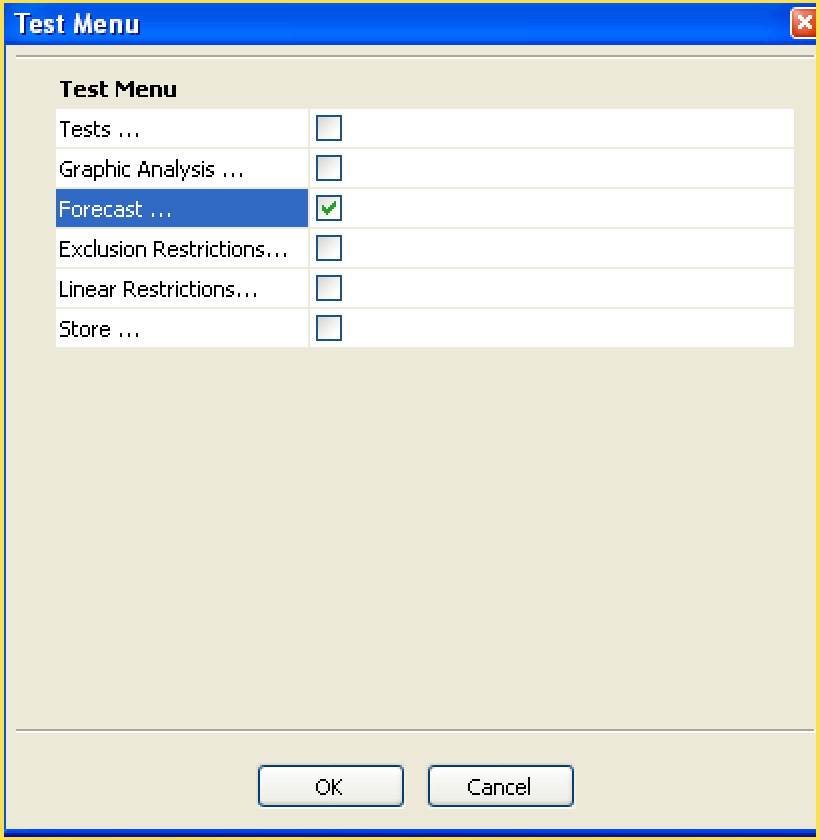
Laurent, S. G@RCH manual, pp. 49-50

Forecasting the Conditional Error Variance

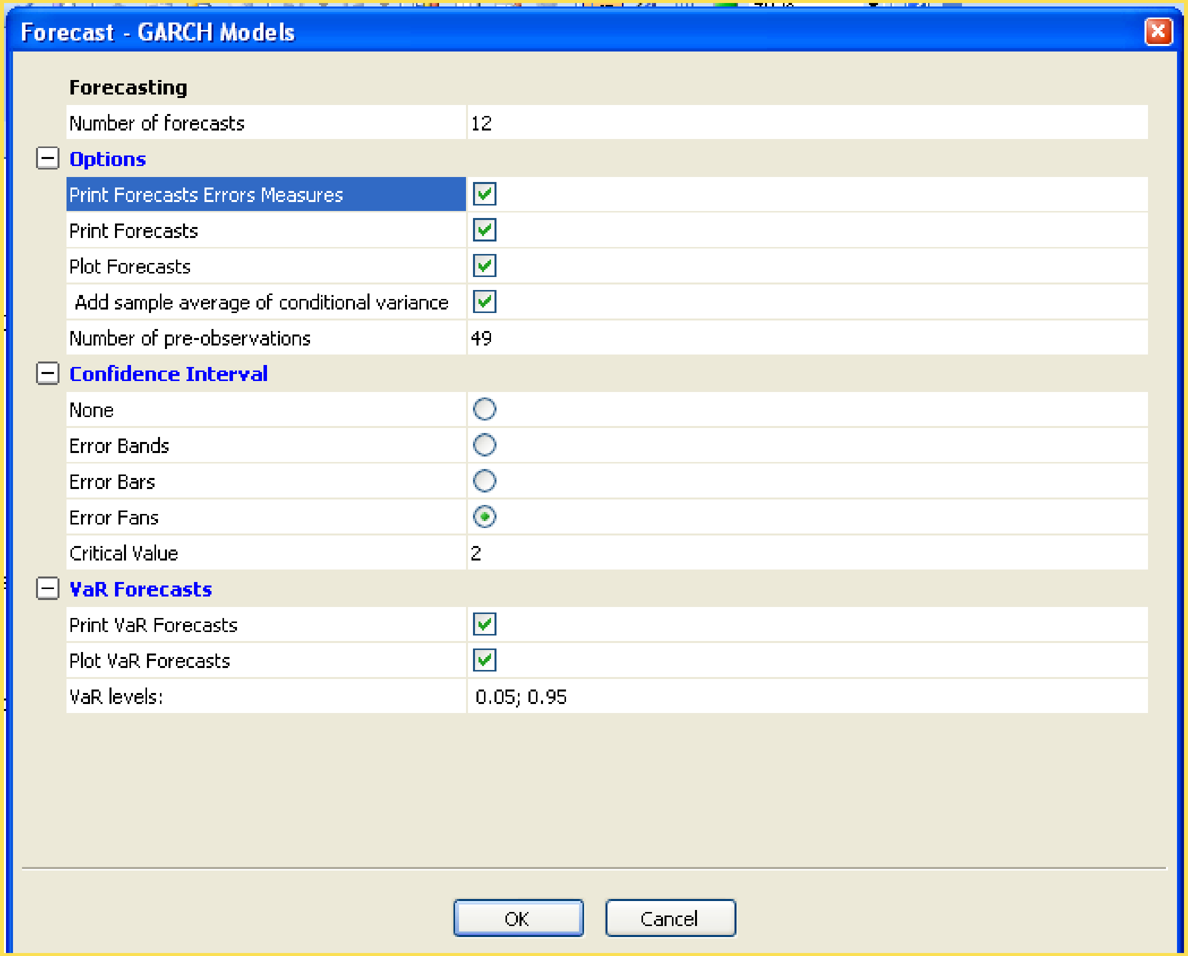
- Suppose we had an ARCH(q) process.

$$\hat{\sigma}^2_{t+h} = \hat{\omega} + \sum_{i=1}^{q} \hat{\alpha}_i^2 \varepsilon^2_{t+h-i|t}$$

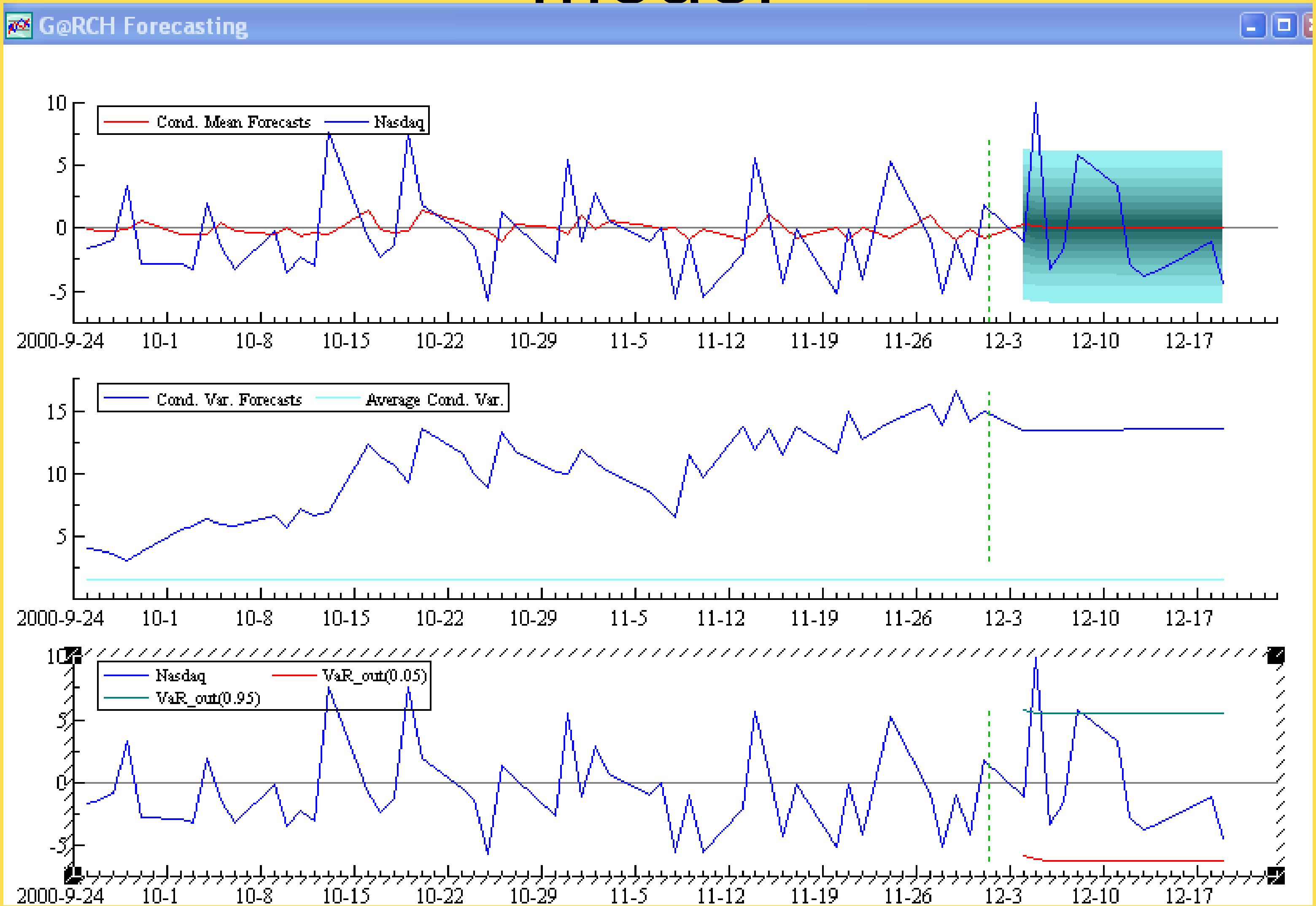
Click on the test icon



Forecast selection



Forecasts graphed from GJR model



Forecasts printed

```
 *******************
 ** VaR FORECASTS **
*******************
Number of Forecasts: 12

Horizon        0.05        0.95
      1      -5.716       8.482
      2      -7.222       6.989
      3      -7.508       6.717
      4      -7.567       6.671
      5      -7.584       6.668
      6      -7.594       6.673
      7      -7.601       6.679
      8      -7.609       6.685
      9      -7.616       6.692
     10      -7.624       6.698
     11      -7.631       6.705
     12      -7.639       6.711
```

Forecast Evaluation

Forecast Evaluation Measures

	Mean	Variance
Mean Squared Error(MSE)	18.41	674
Median Squared Error(MedSE)	10.85	33.91
Mean Error(ME)	-0.5271	4.921
Mean Absolute Error(MAE)	3.664	13.71
Root Mean Squared Error(RMSE)	4.291	25.96
Mean Absolute Percentage Error(MAPE)	.NaN	2.144
Adjusted Mean Absolute Percentage Error(AMAPE)	.NaN	0.3743
Percentage Correct Sign(PCS)	0.25	.NaN
Theil Inequality Coefficient(TIC)	0.9714	0.5777
Logarithmic Loss Function(LL)	.NaN	1.559

Mean Square Error (MSE)

$$MSE(h) = \frac{1}{h}\sum_{t=1}^{H}(\hat{\sigma}_{t+h} - \sigma_{T+t(h)})^2$$

$$where$$

$$h = \ forecast\ horizon\ length$$

$$T = largest\ \ number\ of\ \ in - sample\ \ obs$$

Root mean squared error (RMSE)

$$MSE(h) = \sqrt{\frac{1}{h}\sum_{t=1}^{H}(\hat{\sigma}_{t+h} - \sigma_{T+t(h)})^2}$$

$where$

$h = \text{ } forecast\ horizon\ length$

$T = largest\ \ number\ of\ \ in - sample\ \ obs$

Mean Error (ME)

$$ME = \frac{1}{T}\sum_{t=1}^{T}\left(\hat{\sigma}_t - \sigma_t\right)$$

Mean Absolute Error

$$MAE = \frac{1}{T}\sum_{t=1}^{T} |\hat{\sigma}_t - \sigma_t|$$

Mean Absolute Percentage Error (MAPE)

$$MAPE = \frac{100}{T} \sum_{t}^{T} \frac{|\hat{\sigma}_t - \sigma_t|}{\sigma_t}$$

MAPE tends to exaggerate when the counts are small to begin with.

Adjusted (symmetric) MAPE

- Corrects for asymmetry between actual and forecast values
- Can be interpreted as a percentage error

$$AMAPE = \frac{1}{T - T_1} \sum_{t=T_1}^{T} \left| \frac{x_{t+n} - f_{t,n}}{x_{t+n} + f_{t,n}} \right|$$

$$T = total\ obs\ available$$

$$T - T_1 = holdout\ sample\ used\ for\ forecasting$$

Brooks, C.(1997). Linear and Non-linear Non-Forecastability of High Frequency Exchange Rates, Journal of Forecasting, 16, 125-147.

Symmetric MAPE caveats

- Symmetric MAPE, according to Goodwin and Lawton (2000) IJF (15), 405-408 is not symmetric in that it treats positive and negative errors differently, particularly where they have large absolute values.

Theil's U

$$Theil-U = \frac{1}{T}\frac{\sum_{t=1}^{T}(\hat{\sigma}_t - \sigma_t)^2}{\sum_{t=1}^{T}(\hat{\sigma}_t^{BM} - \sigma_t)^2}$$

BM=baseline model may be a random walk model. This uses another model as a baseline. Scores less than 1.00 are good and those more than 1.00 not so good.

Logarithmic Loss Function

$$LL = \frac{1}{T}\sum_{t=1}^{T}\left[\ln(e_{T-t}^{2}) - \ln(\hat{h}_{T-t})\right]^{2}$$

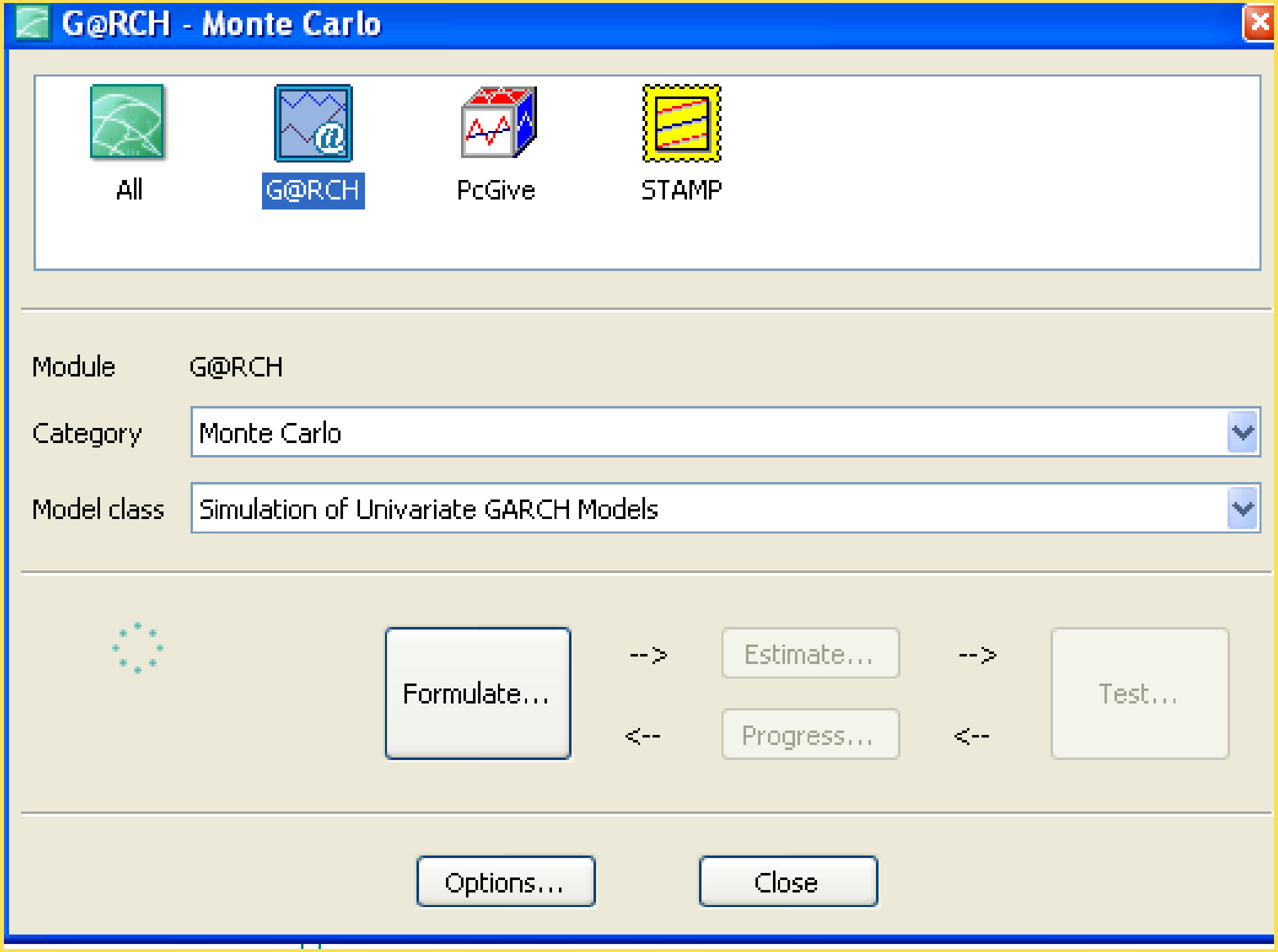
Lopez, J. (1999) Evaluating the Predictive Accuracy of Volatility Models, FRB of San Francisco, p.6.

Forecasting VaR follows shortly

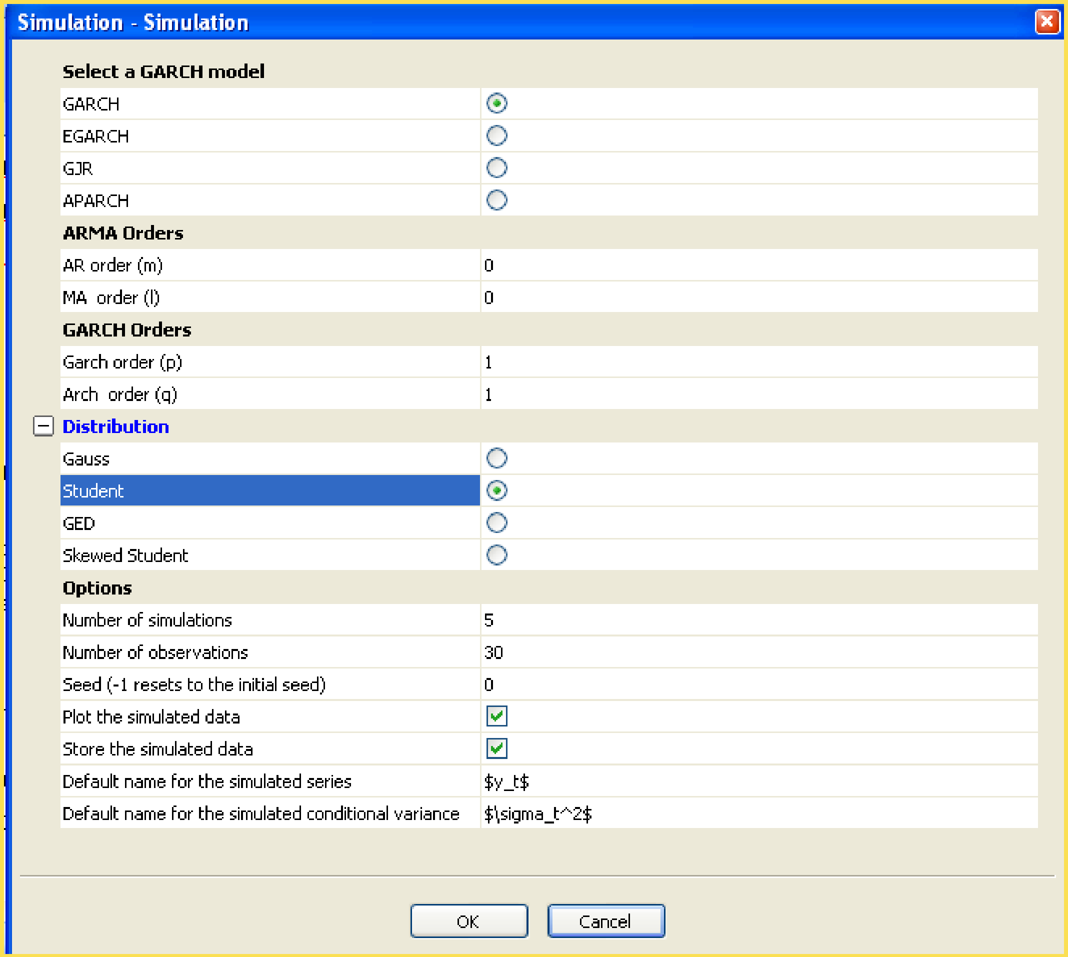
Simulation of CEV confidence intervals

- The model just run can be simulated from the Ox Code.
- The simulations generates multiple replications.
- Means and standard errors can be computed.
- These CEV means and standard errors can be graphed.

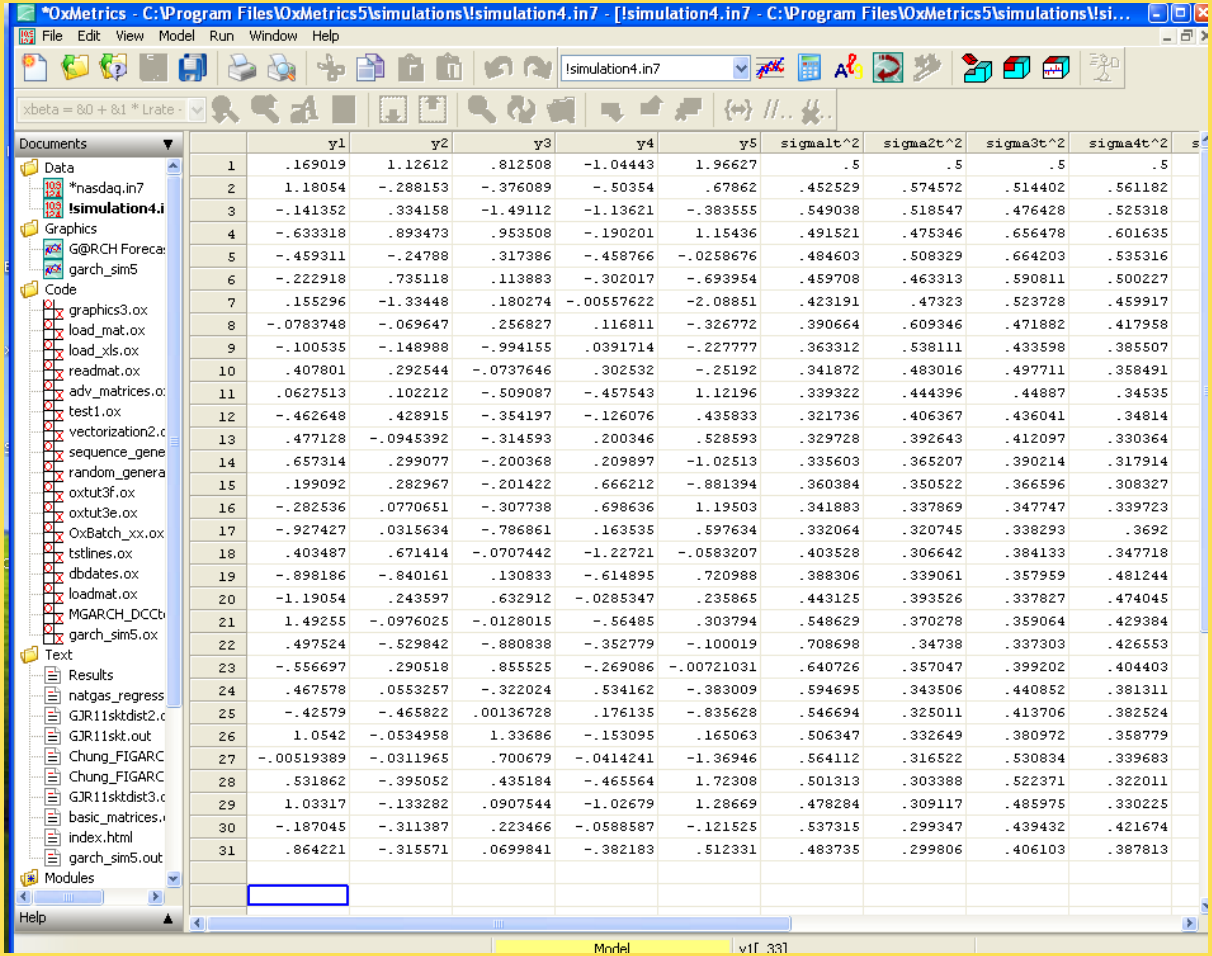
Simulation of GARCH models



Simulation menu



Simulations file created

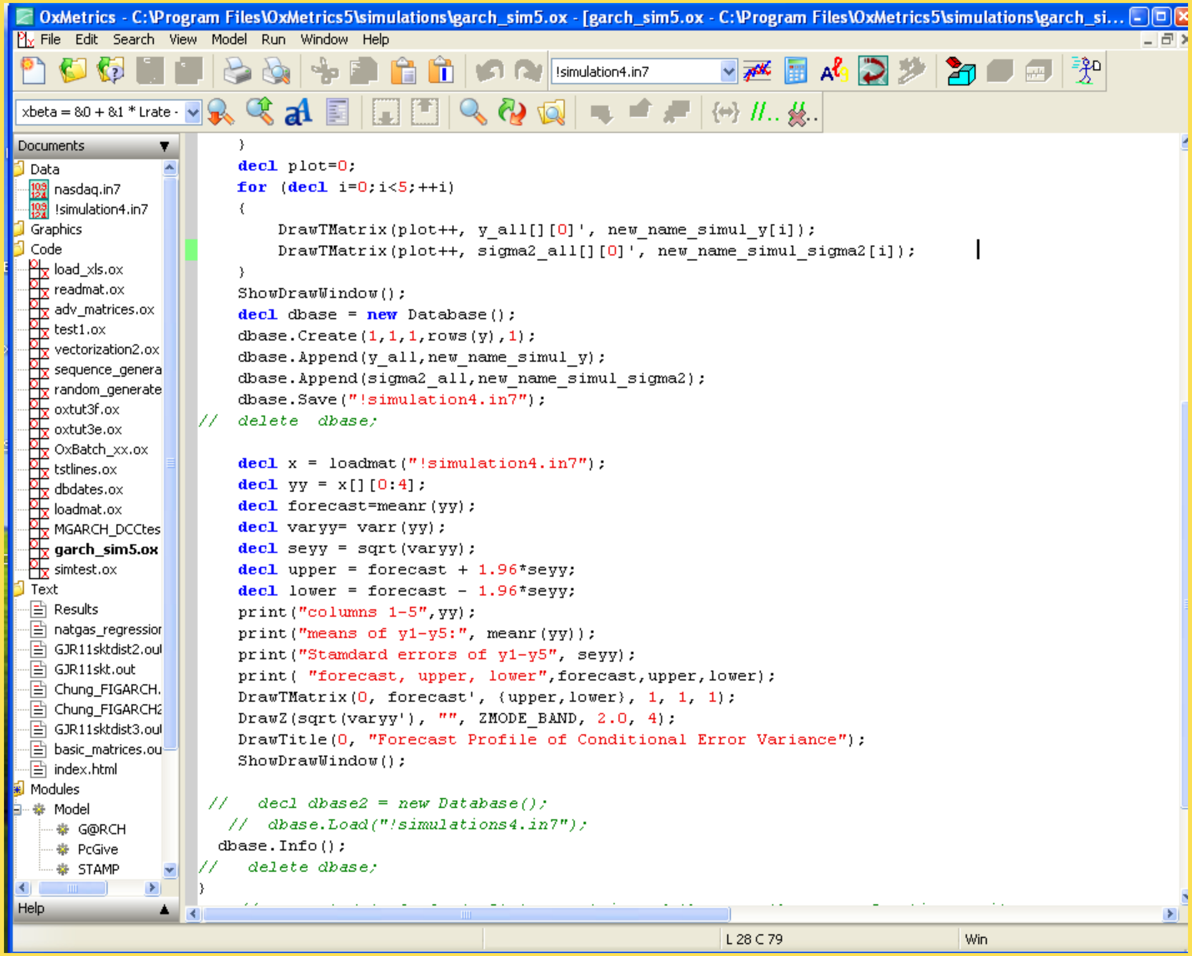


	y1	y2	y3	y4	y5	sigma1t^2	sigma2t^2	sigma3t^2	sigma4t^2
1	.169019	1.12612	.812508	-1.04443	1.96627	.5	.5	.5	.5
2	1.18054	-.288153	-.376089	-.50354	.67862	.452529	.574572	.514402	.561182
3	-.141352	.334158	-1.49112	-1.13621	-.383555	.549038	.518547	.476428	.525318
4	-.633318	.893473	.953508	-.190201	1.15436	.491521	.475346	.656478	.601635
5	-.459311	-.24788	.317386	-.458766	-.0258676	.484603	.508329	.664203	.535316
6	-.222918	.735118	.113883	-.302017	-.693954	.459708	.463313	.590811	.500227
7	.155296	-1.33448	.180274	-.00557622	-2.08851	.423191	.47323	.523728	.459917
8	-.0783748	-.069647	.256827	.116811	-.326772	.390664	.609346	.471882	.417958
9	-.100535	-.148988	-.994155	.0391714	-.227777	.363312	.538111	.433598	.385507
10	.407801	.292544	-.0737646	.302532	-.25192	.341872	.483016	.497711	.358491
11	.0627513	.102212	-.509087	-.457543	1.12196	.339322	.444396	.44887	.34535
12	-.462648	.428915	-.354197	-.126076	.435833	.321736	.406367	.436041	.34814
13	.477128	-.0945392	-.314593	.200346	.528593	.329728	.392643	.412097	.330364
14	.657314	.299077	-.200368	.209897	-1.02513	.335603	.365207	.390214	.317914
15	.199092	.282967	-.201422	.666212	-.881394	.360384	.350522	.366596	.308327
16	-.282536	.0770651	-.307738	.698636	1.19503	.341883	.337869	.347747	.339723
17	-.927427	.0315634	-.786861	.163535	.597634	.332064	.320745	.338293	.3692
18	.403487	.671414	-.0707442	-1.22721	-.0583207	.403528	.306642	.384133	.347718
19	-.898186	-.840161	.130833	-.614895	.720988	.388306	.339061	.357959	.481244
20	-1.19054	.243597	.632912	-.0285347	.235865	.443125	.393526	.337827	.474045
21	1.49255	-.0976025	-.0128015	-.56485	.303794	.548629	.370278	.359064	.429384
22	.497524	-.529842	-.880838	-.352779	-.100019	.708698	.34738	.337303	.426553
23	-.556697	.290518	.855525	-.269086	-.00721031	.640726	.357047	.399202	.404403
24	.467578	.0553257	-.322024	.534162	-.383009	.594695	.343506	.440852	.381311
25	-.42579	-.465822	.00136728	.176135	-.835628	.546694	.325011	.413706	.382524
26	1.0542	-.0534958	1.33686	-.153095	.165063	.506347	.332649	.380972	.358779
27	-.00519389	-.0311965	.700679	-.0414241	-1.36946	.564112	.316522	.530834	.339683
28	.531862	-.395052	.435184	-.465564	1.72308	.501313	.303388	.522371	.322011
29	1.03317	-.133282	.0907544	-1.02679	1.28669	.478284	.309117	.485975	.330225
30	-.187045	-.311387	.223466	-.0588587	-.121525	.537315	.299347	.439432	.421674
31	.864221	-.315571	.0699841	-.382183	.512331	.483735	.299806	.406103	.387813

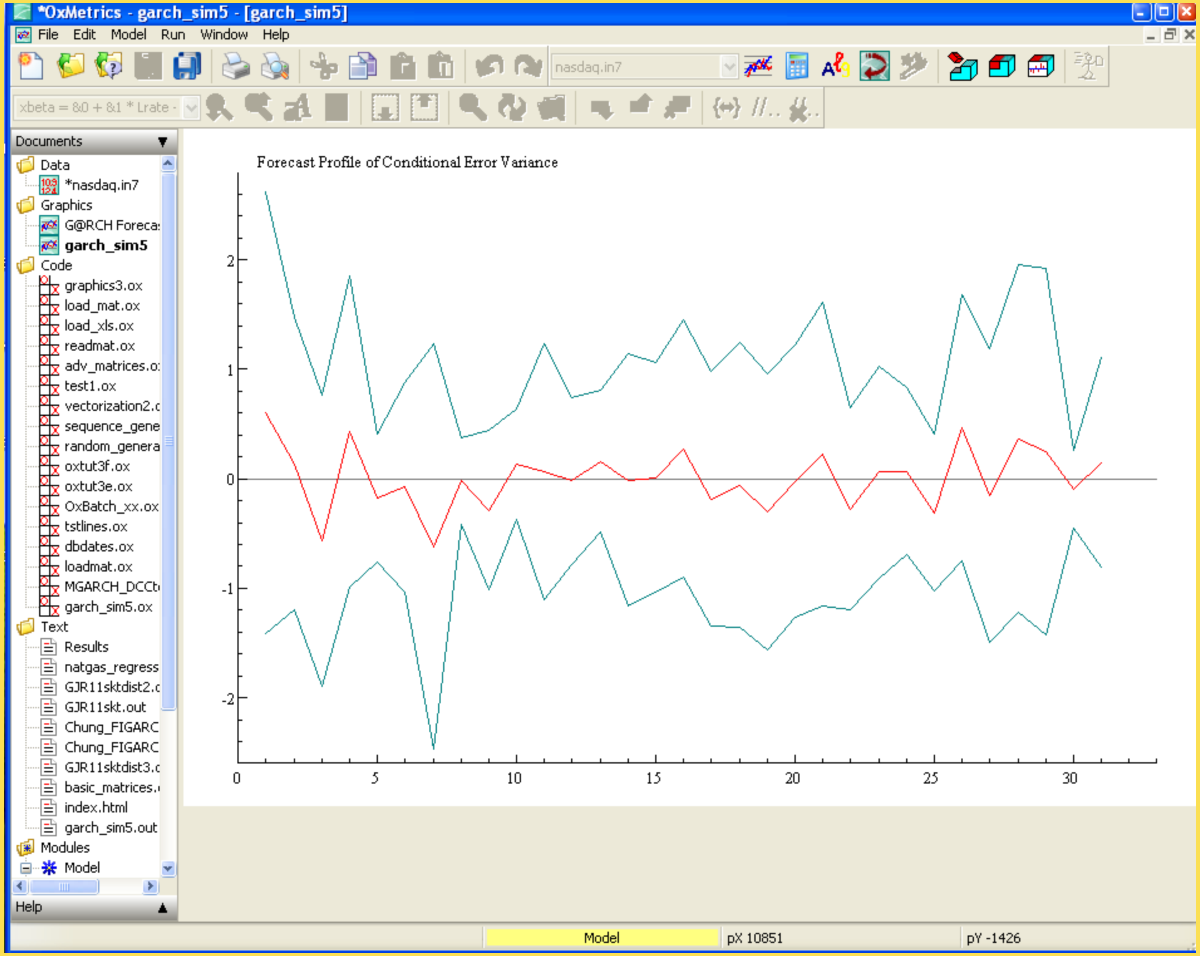
Alt-O invokes the Ox Code of the model just run

```
#include <oxstd.h>
#import <packages/Garch5/garch>
#include <oxdraw.h>
#import <database>
#import <simulations/>
main()
{
    //--- Ox code for G@RCH( 0)
    decl model = new Garch();

    decl z,eps,sigma2,y,y_all=<>,sigma2_all=<>;
    decl new_name_simul_y=new array[5];
    decl new_name_simul_sigma2=new array[5];
    for (decl i=0;i<5;++i)
    {
        z=rann(31,1);
        model.Simulate_GARCH(0.05,<0.1>,<0.8>, z, 0, &eps, &sigma2);
        y=eps+0.01;
        y_all~=y;
        sigma2_all~=sigma2;
        new_name_simul_y[i]=sprint("y",i+1);
        new_name_simul_sigma2[i]=sprint("sigma",i+1,"t^2");
    }
    decl plot=0;
    for (decl i=0;i<5;++i)
    {
        DrawTMatrix(plot++, y_all[][0]', new_name_simul_y[i]);
        DrawTMatrix(plot++, sigma2_all[][0]', new_name_simul_sigma2[i]);
    }
    ShowDrawWindow();
    decl dbase = new Database();
    dbase.Create(1,1,1,rows(y),1);
    dbase.Append(y_all,new_name_simul_y);
    dbase.Append(sigma2_all,new_name_simul_sigma2);
    dbase.Save("!simulation4.in7");
//  delete  dbase;
```

Remainder of Ox code for simulation



Graphed simulated confidence intervals around the Conditional Error Variance



Outlier Modeling

- Mean model outliers
- Variance model outliers
- Outliers in both mean and variance model may be designated.
- These can be important in model fitting.

Value-at-Risk

- Normal APARCH (1,1)

$$\sigma_t^{\delta} = \omega + a_1(|\varepsilon_{t-1}| - \gamma_1 \varepsilon_{t-1})^{\delta} + b_1 \sigma_{t-1}^{\delta}$$

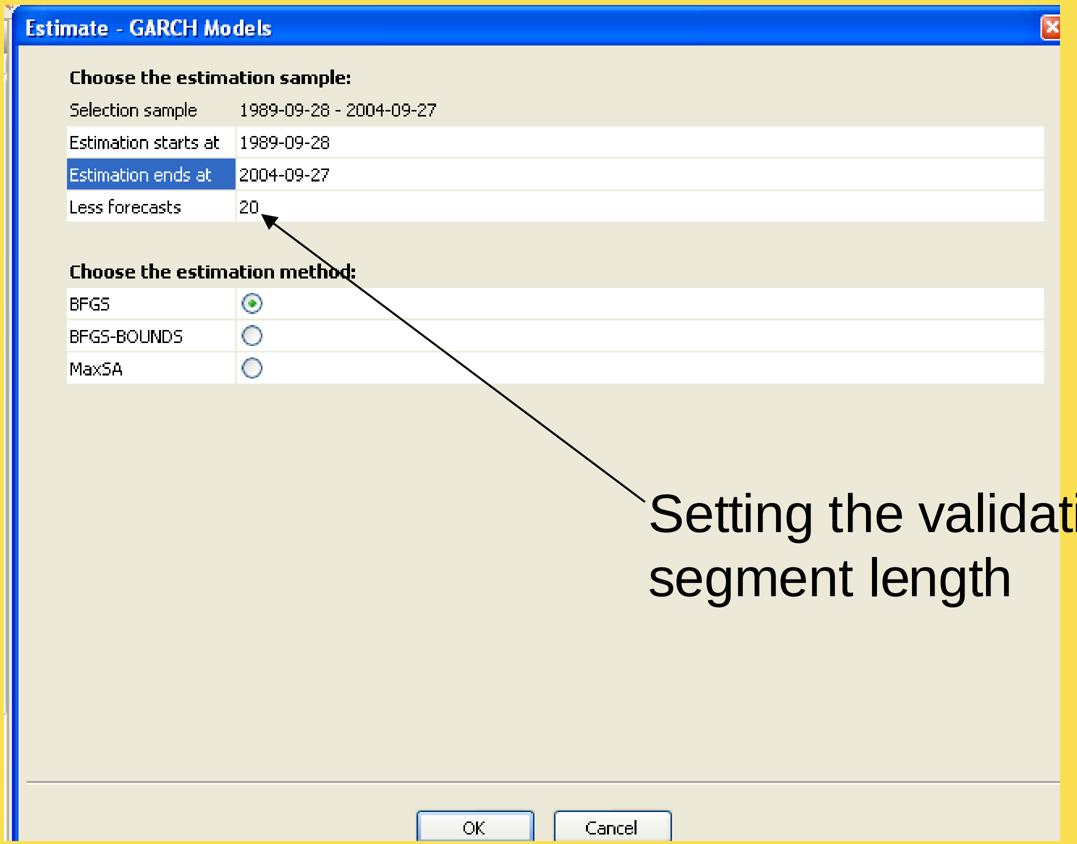
- APARCH has long memory capabilities and threshold capabilities built in. Leverage effects are captured.
- Is usually used with a skewed t distribution. In this case I use an APARCH(1,1) with a t distribution to generate the Value at Risk. It does handle the fat-tails but in this case there is no appreciable asymmetry.

Value-at-Risk

- In-sample
  - Models are tested at α and (1- α) levels for both long and short positions at various VaR quantiles .
  - Graphical output is available here.
  - The failure rate is indicated by number of times absolute value > forecasted VaR.
  - Kupiec test is available
  - Dynamic regression quantile is available.
  - Expected shortfall for long and short positions

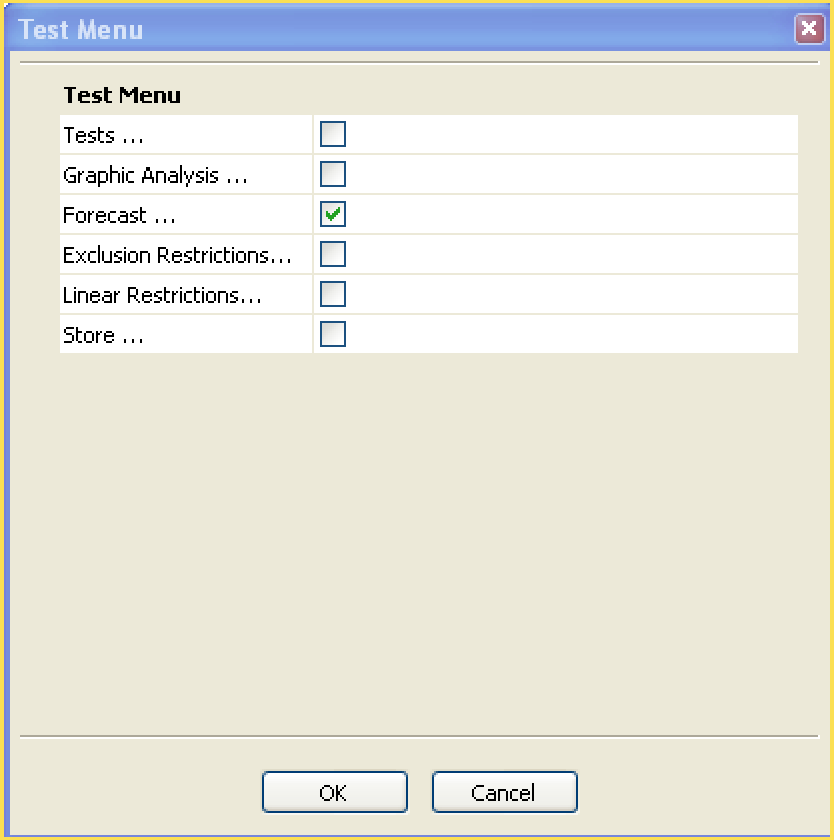
Out-of-sample VaR

- Backtesting on the estimation sample
- Out-of-sample length defined by user

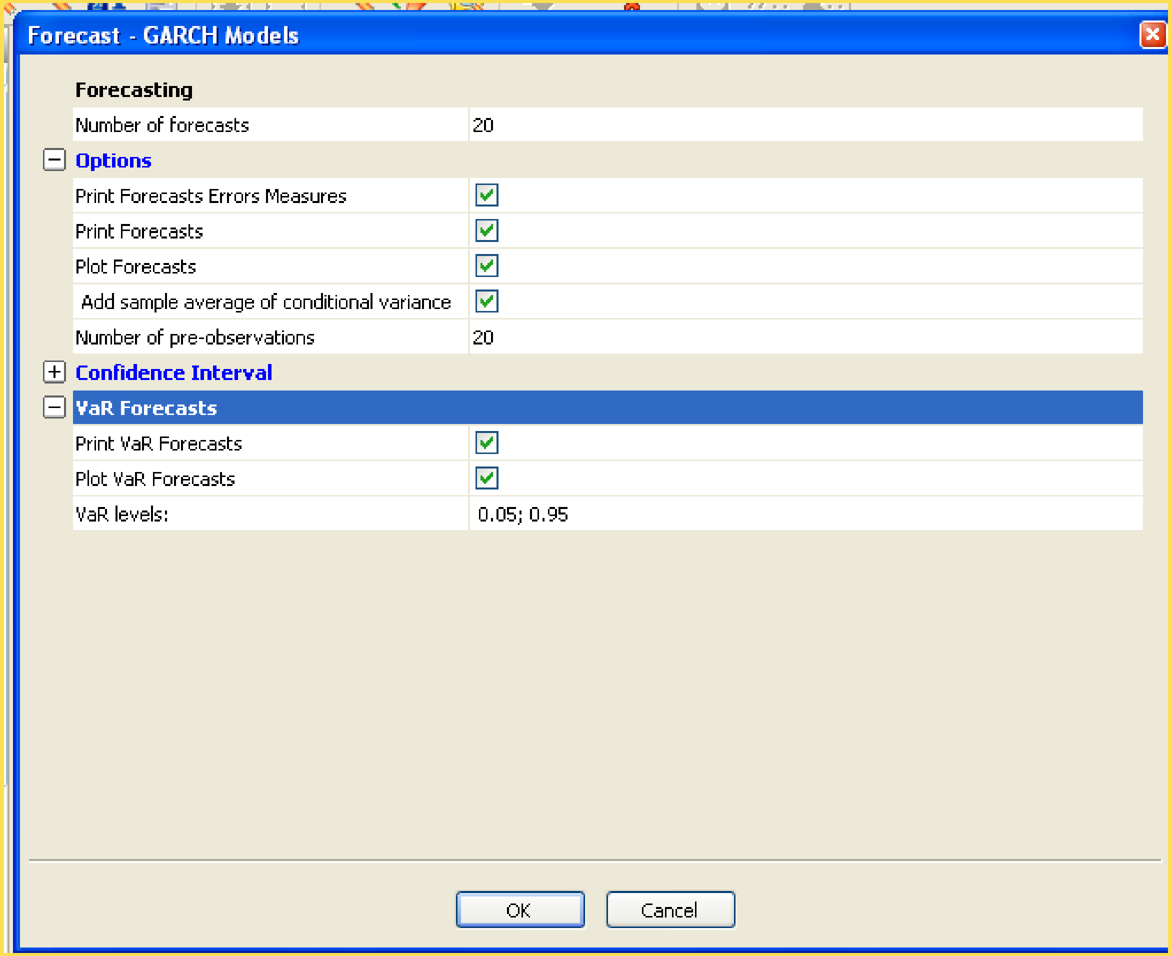


Setting the validation segment length

Opt for Forecasts



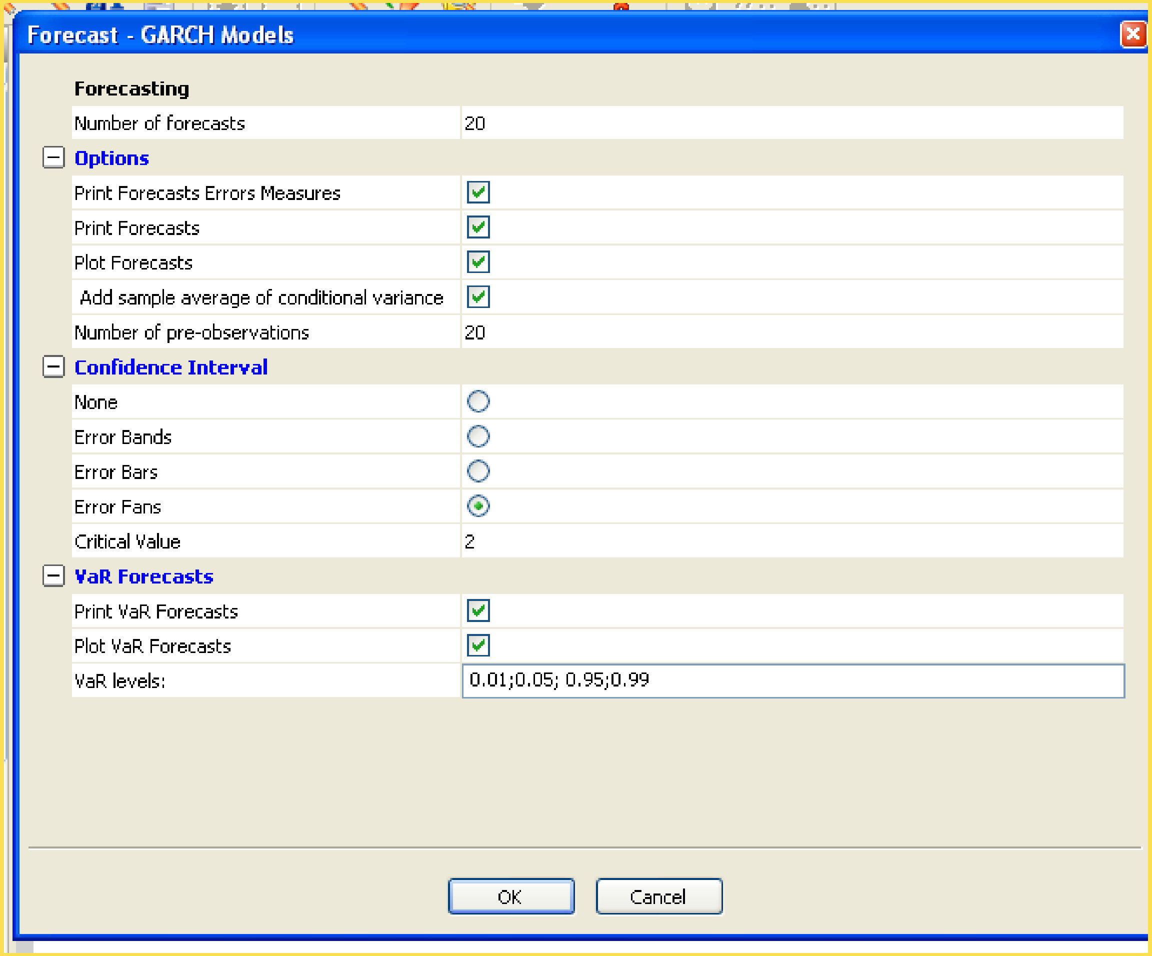
Set the VaR out-of-sample horizon



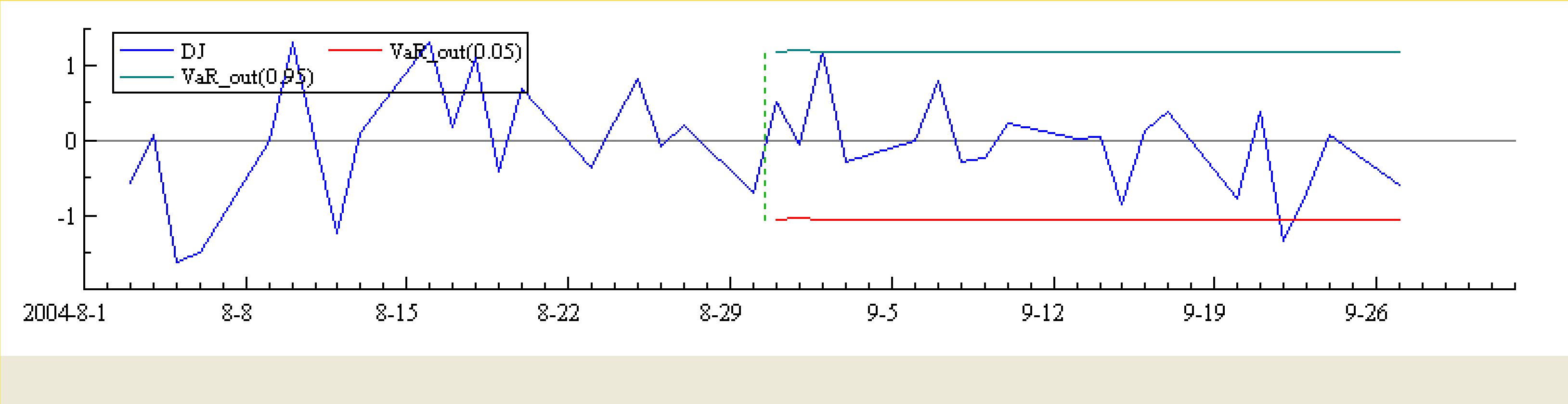
Printout of VaR Forecasts

```
 *******************
** VaR FORECASTS **
*******************
Number of Forecasts: 20

Horizon        0.01        0.05        0.95        0.99
      1      -1.723       -1.06       1.176       1.839
      2      -1.702      -1.039       1.197        1.86
      3      -1.722      -1.059       1.177        1.84
      4      -1.722      -1.059       1.177        1.84
      5      -1.722      -1.059       1.177        1.84
      6      -1.722      -1.059       1.177        1.84
      7      -1.722      -1.059       1.177        1.84
      8      -1.722      -1.059       1.177        1.84
      9      -1.722      -1.059       1.177        1.84
     10      -1.722      -1.059       1.177        1.84
     11      -1.722      -1.059       1.177        1.84
     12      -1.722      -1.059       1.177        1.84
     13      -1.722      -1.059       1.177        1.84
     14      -1.722      -1.059       1.177        1.84
     15      -1.722      -1.059       1.177        1.84
     16      -1.722      -1.059       1.177        1.84
     17      -1.722      -1.059       1.177        1.84
     18      -1.722      -1.059       1.177        1.84
     19      -1.722      -1.059       1.177        1.84
     20      -1.722      -1.059       1.177        1.84
---------------
```

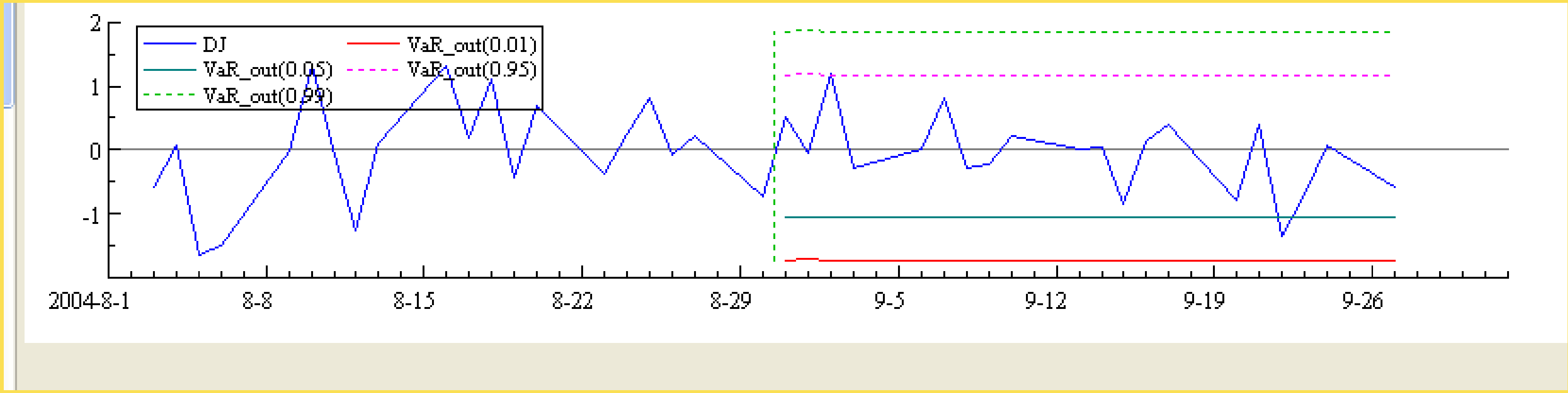
Requesting More VaR levels



Graphical Forecast of out-of-sample VaR



Graphical Forecast of out-of-sample VaR



Long Memory Models

- APARCH (Ding, Engle, and Granger,1993)
- FIGARCH-Baillie, Bollerslev, and Mikkelsen(BBM)
- FIGARCH-Chung
- FIAPARCH (Tse, 1998)
- FIAPARCH-Chung
- FIEGARCH (Bollerslev and Mikkelsen, 1996)
- Davidson's Hyperbolic GARCH

Long-Memory Processes

Fractional differencing for long memory processes

$$(1-L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(d+1)}{\Gamma(k+1)\Gamma(d-k+1)} L^k$$

where

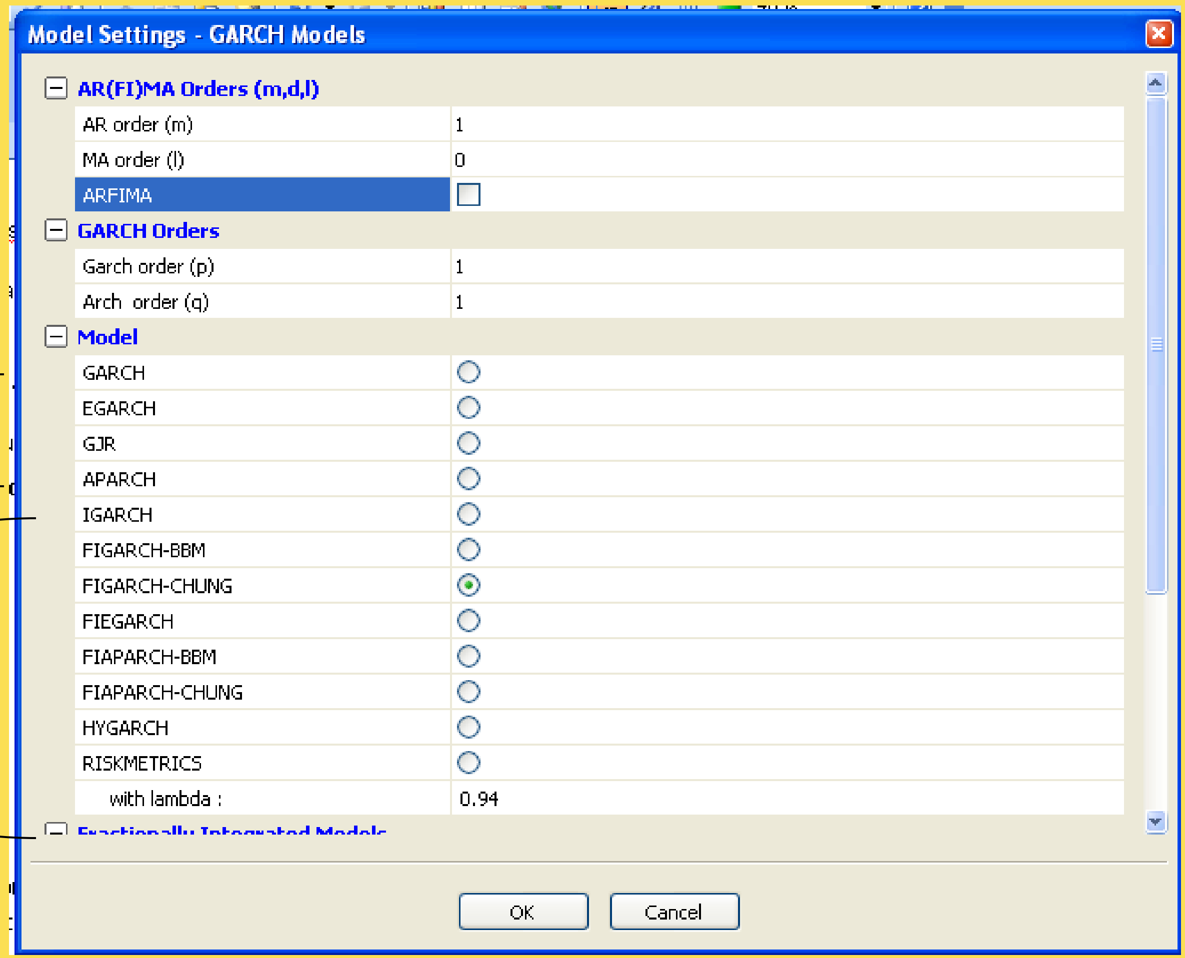
$$\Gamma(n) = \text{gamma function } (n-1)!$$

We substitute this function for (1-L) in FIGARCH, etc.

Long-Memory Models

- We run the basic descriptives test on it
  - And find that it has long memory with a GPH
  - d = .2885 with p = 0.0000.
  - Therefore we try a long-memory model.
  - A FIGARCH - Chung model

Asymmetric and Long Memory models



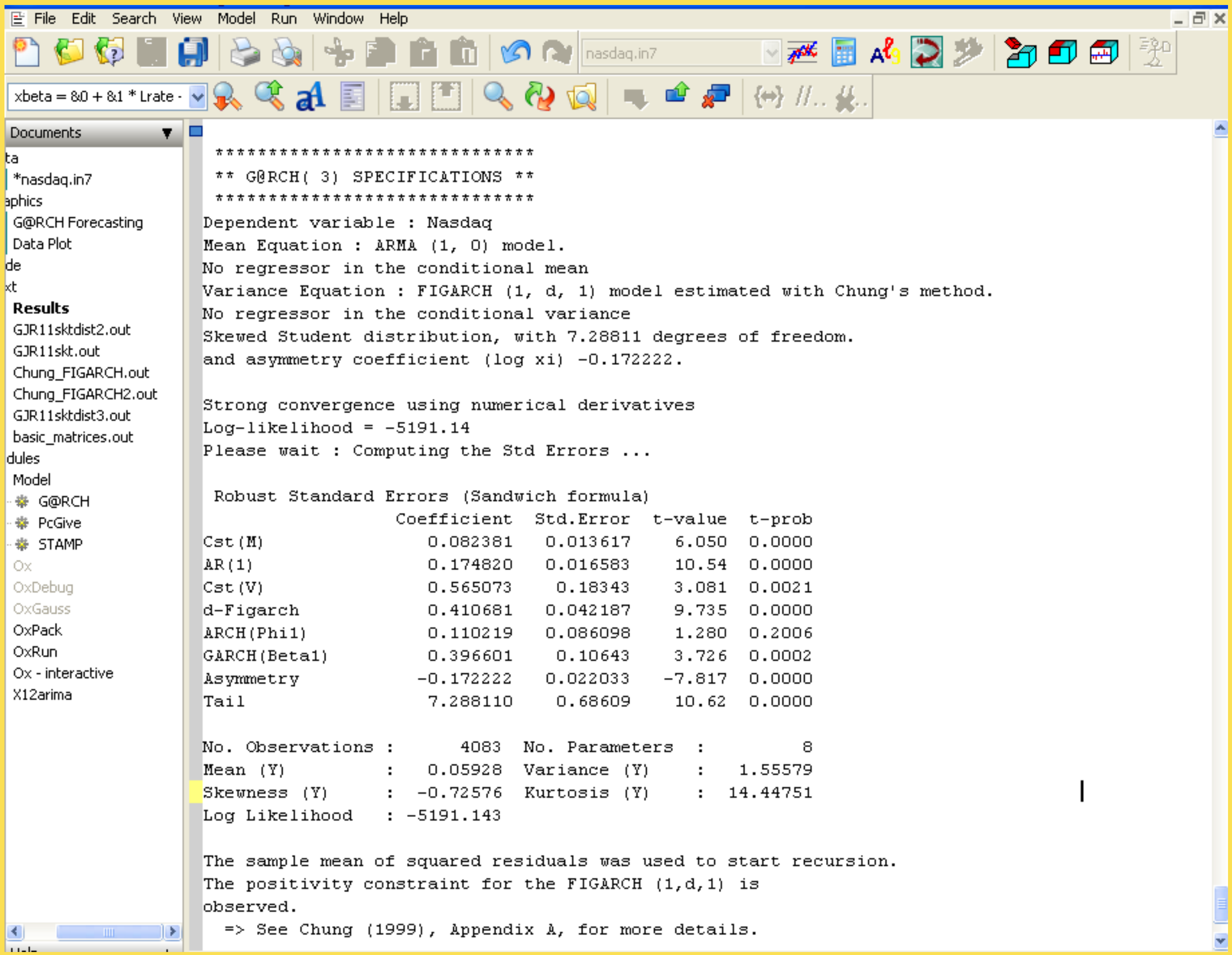
asymmetric

Long memory

Ar(1) -Chung's Method with normal distribution

```
 ******************************
 ** G@RCH( 5) SPECIFICATIONS **
 ******************************
Dependent variable : Nasdaq
Mean Equation : ARMA (1, 0) model.
No regressor in the conditional mean
Variance Equation : FIGARCH (1, d, 1) model estimated with Chung's method.
No regressor in the conditional variance
Normal distribution.

Strong convergence using numerical derivatives
Log-likelihood = -5385.77
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.088338   0.016143    5.472  0.0000
AR(1)                0.197900   0.018280    10.83  0.0000
Cst(V)               0.821616    0.27665    2.970  0.0030
d-Figarch            0.358937   0.048296    7.432  0.0000
ARCH(Phi1)           0.045390    0.16180   0.2805  0.7791
GARCH(Beta1)         0.240098    0.18513    1.297  0.1947

No. Observations :      4093  No. Parameters   :         6
Mean (Y)         :   0.05517  Variance (Y)     :   1.59189
Skewness (Y)     :  -0.74128  Kurtosis (Y)     :  14.25531
Log Likelihood   : -5385.775

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the FIGARCH (1,d,1) is
observed.
  => See Chung (1999), Appendix A, for more details.
```

AR(1) Chung Model sk(t)



```
 ******************************
 ** G@RCH( 3) SPECIFICATIONS **
 ******************************
Dependent variable : Nasdaq
Mean Equation : ARMA (1, 0) model.
No regressor in the conditional mean
Variance Equation : FIGARCH (1, d, 1) model estimated with Chung's method.
No regressor in the conditional variance
Skewed Student distribution, with 7.28811 degrees of freedom.
and asymmetry coefficient (log xi) -0.172222.

Strong convergence using numerical derivatives
Log-likelihood = -5191.14
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.082381   0.013617    6.050  0.0000
AR(1)                0.174820   0.016583    10.54  0.0000
Cst(V)               0.565073    0.18343    3.081  0.0021
d-Figarch            0.410681   0.042187    9.735  0.0000
ARCH(Phi1)           0.110219   0.086098    1.280  0.2006
GARCH(Beta1)         0.396601    0.10643    3.726  0.0002
Asymmetry           -0.172222   0.022033   -7.817  0.0000
Tail                 7.288110    0.68609    10.62  0.0000

No. Observations :      4083  No. Parameters  :         8
Mean (Y)         :   0.05928  Variance (Y)    :   1.55579
Skewness (Y)     :  -0.72576  Kurtosis (Y)    :  14.44751
Log Likelihood   : -5191.143

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the FIGARCH (1,d,1) is
observed.
  => See Chung (1999), Appendix A, for more details.
```

Hyperbolic Garch (James Davidson)

- The generalized hyperbolic distribution was discovered by Barndorff-Neilson(1977) researching wind-blown sand.
- This distribution can be skewed and captures asymmetric effects that normal distributions cannot.
- This distribution describes long-memory processes.

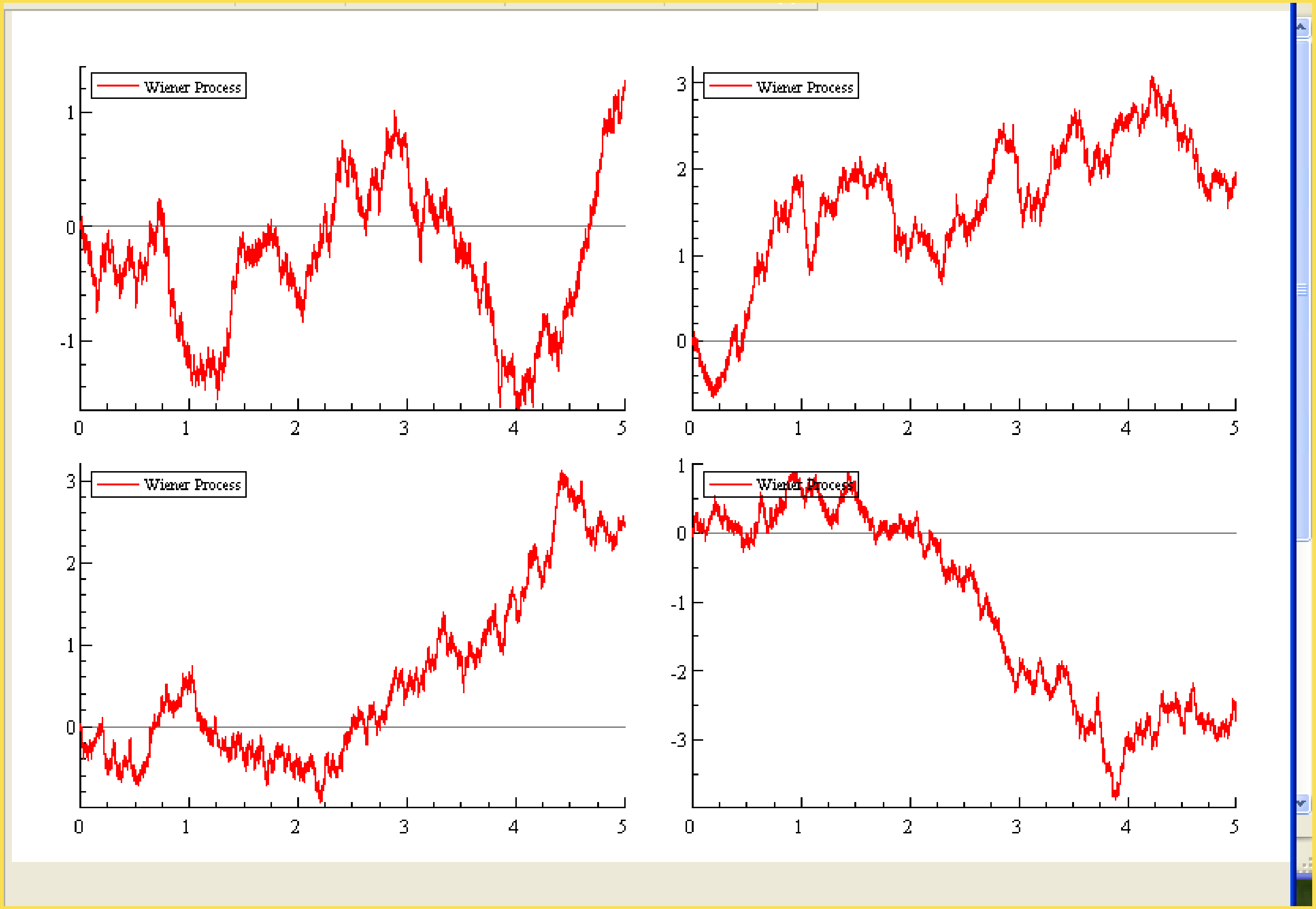
Continuous time Diffusion Models

- Brownian motion simulation
- Diffusion models
- Diffusion models with jumps
- Microstructure noise with jumps

Ox Programs for Realized and Integrated Volatility diffusion models

```
#include <oxstd.h>
#include <oxdraw.h>
#import <packages/garch5/garch>

main()
{
    decl obs_per_day=2880; //2880
    decl number_days=510;
    decl remove_first_days=10;
    decl m=obs_per_day*number_days;
    decl Delta=1/obs_per_day;
    decl select_every_obs=10; //10
    decl theta=0.035;
    decl omega=0.635;
    decl lambda=0.296;
    decl p0=1;
    decl s20=0.1;
    decl P,Spot_vol;

    decl garchobj = new Garch();
    garchobj.Simul_Continuous_GARCH(p0,s20,m, Delta,theta,omega,lambda,&P,&Spot_vol);    // SIMULA
    // Remove the first 'remove_first_days' observations
    if (remove_first_days>0)
    {
        P=P[remove_first_days*obs_per_day:];
        Spot_vol=Spot_vol[remove_first_days*obs_per_day:];
        number_days-=remove_first_days;
        m=obs_per_day*number_days;
    }

    // Compute the Integrated volatility
    decl IV=sumr(reshape(Spot_vol,number_days,obs_per_day).*Delta);

    // Compute the 5-min prices and daily prices
    decl sel = reshape(zeros(select_every_obs-1,1)|1,m,1);
    decl P_5min=selectifr(P,sel);
```

Ox can simulate continuous time Brownian Motion processes



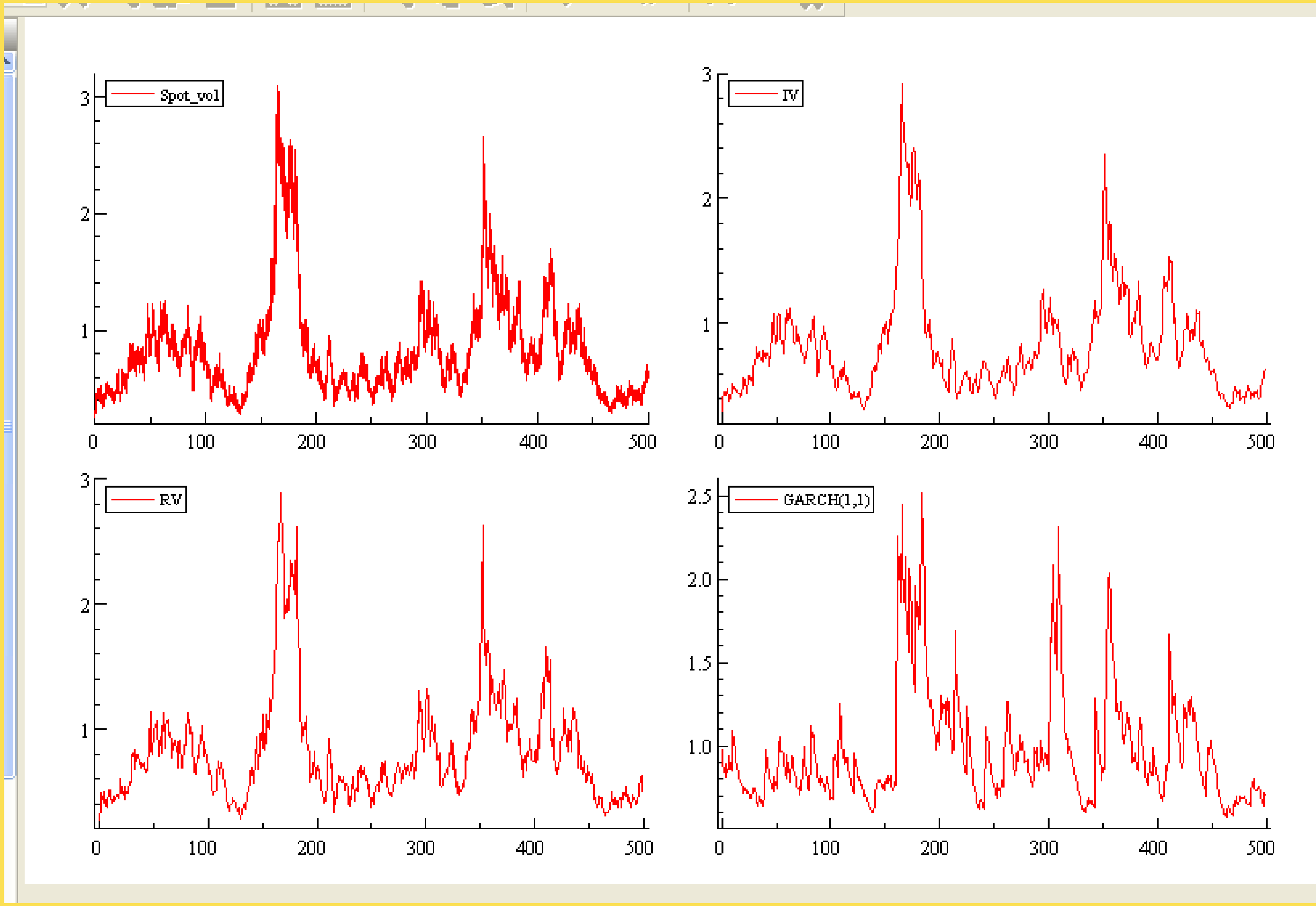
GARCH type output

```
 ********************
 ** SPECIFICATIONS **
 ********************
Dependent variable : Daily returns
Mean Equation : ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation : GARCH (1, 1) model.
No regressor in the conditional variance
Normal distribution.

Strong convergence using numerical derivatives
Log-likelihood = -689.927
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.030386   0.040286   0.7543  0.4510
Cst(V)               0.086647   0.049648    1.745  0.0816
ARCH(Alpha1)         0.084538   0.033322    2.537  0.0115
GARCH(Beta1)         0.826288   0.067380    12.26  0.0000

No. Observations :        500  No. Parameters  :         4
Mean (Y)         :    0.02944  Variance (Y)    :   0.97345
Skewness (Y)     :    0.04487  Kurtosis (Y)    :   4.30813
Log Likelihood   :   -689.927  Alpha[1]+Beta[1]:   0.91083

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is alpha[L]/[1 - beta(L)] >= 0.
The unconditional variance is 0.971656
The conditions are alpha[0] > 0, alpha[L] + beta[L] < 1 and alpha[i] + beta[i] >= 0.
  => See Doornik & Ooms (2001) for more details.
The condition for existence of the fourth moment of the GARCH is observed.
The constraint equals 0.843896 and should be < 1.
  => See Ling & McAleer (2001) for details.
```

Mean Spot and Integrated Volatility
Mean GARCH volatility

```
Estimated Parameters Vector :
 0.030386; 0.086647; 0.084538; 0.826288

Elapsed Time : 0.125 seconds (or 0.00208333 minutes).

Mean Spot vol: 0.82768
Mean IV: 0.82768
Mean GARCH vol: 0.975741
Mean squared daily returns: 0.974312
```

Graphical Output



Other Ox Diffusion Models

- Other diffusion models include diffusion models for estimation of realized volatility with jumps.
- Lee-Mykland's statistical test for detecting jumps at ultra-high-frequency.
- Estimation of integrated volatility with jumps.
- estimation of microstructure noise.
- Estimation of intraday seasonality with flexible Fourier functional form filter.

Multivariate GARCH

- Engle and Kroner (1995) Vec Model
- Baba,Engle, Kraft, Kroner (BEKK) models
  - Scalar
  - Diagonal
- RiskMetrics MGARCH
- Factor GARCH
  - Carol Alexander's Orthogonal GARCH
  - GOGARCH Generalized Orthogonal GARCH (ML, NLS)

Vec Model
(Engle and Kroner, 1995)

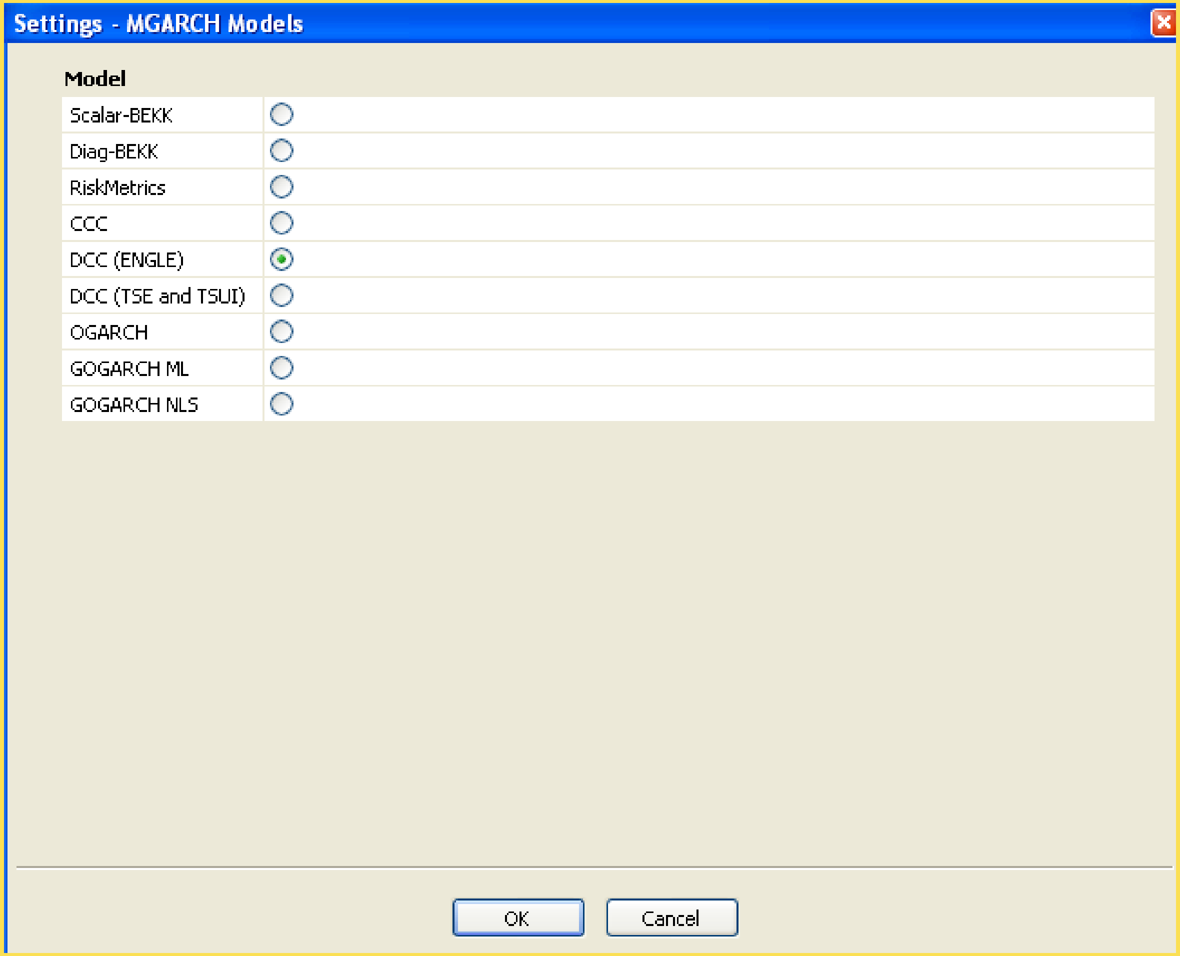
$$Vec(H_t) = vec(\Omega) + Avec(r_{t-1}r_{t-1}') + Bvec(H_{t-1})$$

$where\ A\ and\ B\ are\ n^2 x n^2 matrices\ with\ structure$
$following\ from\ symmetry\ of\ H_t$
$vec = column\ stacking\ operator$

$with\ variance\ targeting,\ vec(\Omega) = (I - A - B)vec(S)$

$$where\ S = \frac{1}{T}\sum_t (r_t r_t')$$

Multivariate GARCH menu



BEKK(p,q) Model
Baba, Engle, Kraft, and Kroner (1995)

$$H_t = C'C + \sum_{i=1}^{q} A_i' \varepsilon_{t-1} \varepsilon_{t-1}' A_i + \sum_{j=1}^{p} G_j' H_{t-j} G_j$$

C,A, and G are nxn, but C is upper triangular

Problem-number of parameters

- ARCH and GARCH BEKK(1,1) models have N(5*N+1)/2 parameters. This is a lot.
- To reduce the number of parameters, constraints have to be imposed.
- The curse of dimensionality can slow down or cause the model to fail converge.

Assumptions

Kronecker product

This model is covariance stationary if

$$\sum_{i=1}^{q} a_{nn,i}^{2} + \sum_{j=1}^{p} g_{nn,j}^{2} < 1$$

When it exists, the unconditional variance matrix $\Sigma \equiv E(H_t)$ *of the BEKK model* =

$$vec(\Sigma) = \left[I_{N^2} - \sum_{i=1}^{q} (A_i \otimes A_i)' - \sum_{j=1}^{p} (G_j \otimes G_j)' \right]^{-1}$$

Kronecker Product

Let A be an mxn matrix and B be a pxq matrix,

then the Kronecker product $A \otimes B$ *is an mpxnq matrix*

$$\begin{pmatrix} a_{11}B \ldots a_{1n}B \\ . \qquad\qquad . \\ . \qquad\qquad . \\ . \qquad\qquad . \\ a_{m1}B \ldots a_{mn}B \end{pmatrix} \quad \text{and if} \quad A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, \ \text{then}$$

$$A \otimes B = \begin{pmatrix} A_{11}B & A_{12}B \\ A_{21}B & A_{22}B \end{pmatrix}$$

Variance Targeting (Engle and Mezrich, 1996)

- An estimate of the unconditional covariance matrix was obtained by variance targeting.
- This reduces the number of parameters that needs to be estimated.
- In the BEKK model, we replace C'C by

$$unvec[I_{N^2} - \sum_{t=1}^{q}(A \otimes A)' - \sum_{t=1}^{p}(G \otimes G)']\hat{\Sigma}$$

$where$

$unvec\ is\ the\ column\ unstacking\ operator$

$\hat{\Sigma} = unconditional\ vcv\ of\ \varepsilon_t$

Diagonal BEKK

- Matrices C and G are diagonal to restrict the number of parameters.

Scalar BEKK

- Another way to reduce the number of parameters is to run a Scalar BEKK.
- Matrices A and G are matrices of ones multiplied by a scalar.

RiskMetrics MGARCH (J.P. Morgan, 1996)

$$H_t = (1-\lambda)\varepsilon_{t-1}\varepsilon_{t-1}^{'} + \lambda H_{t-1}$$

or

$$Ht = \frac{(1-\lambda)}{(1-\lambda)^{t-1}} \sum_{t=1}^{t-1} \lambda^{i-1} \varepsilon_{t-1}\varepsilon_{t-1}^{'}$$

where the decay factor

$$0 < \lambda < 1$$

$\lambda = .94$ *for daily data*

$\lambda = .97$ *for monthly data*

“Orthogonal GARCH”

by Carol Alexander(2001), Orthogonal GARCH in Alexander, Carol. (ed). *Mastering Risk Vol. 2: Applications*, Financial Times, pp.24-38

- Suppose you have: T obs, K asset or risk factors is summarized by TxK matrix Y.
- You can generate factor GARCH where the components are univariate GARCH models. We begin with Principal Components Analysis.
- PCA will yield up to k components.

- Procedure
  - 1 standardize the series in TxK matrix X.

Orthogonal GARCH procedure-cont'd

- X represents the same variables in Y.
- Standardize the columns in X so that they have mean=0 and std dev=1, so if ith
- Risk factor or asset return in system is y, then the normalized variables are

$$x_i = (y_i - \mu_i) / \sigma_i$$

$$where \;\; \mu = mean$$

$$\sigma = std\ dev\ of\ i.$$

Orthogonal GARCH procedure-cont'd

- Construct the Sum of squares and cross-products matrix, $R=X'X$.
- Solve Canonical equation of $(R-\Lambda I)W=0$ for eigenvalue-eigenvector decomposition.
- Solve for W = eigenvectors of $X'X$
- Solve for Λ=diagonal matrix of eigenvalues, ordered by decreasing magnitude.

Orthogonal GARCH procedure-cont'd

- The principal components of Y are given by the TxK matrix P = XW.
- X'XW=WΛ.
- P'P=W'X'XW=W'W Λ but because W=orthogonal matrix, W'W=I so
- P'P= Λ,the diagonal matrix of eigenvalues, Variance of the ith component equals the ith eigenvalue of X'X.
- The standardized residuals $\varepsilon_t = H_t^{-1}(y_t - \mu)$

Orthogonal GARCH procedure cont'd

$$H_t = Var_{t-1}(\varepsilon_t) = V^{1/2} V_t V'^{1/2}$$

OGARCH(1,1,m)
Alexander and Chibumba(1997)

$$y_t = u_t + \varepsilon_t$$

$$\varepsilon_t = V^{1/2} u_t$$

$$u_t = Z_m f_t$$

$$\varepsilon_t = V^{1/2} Z_m f_t$$

where

$$V = diagonal(v_1, v_2, ... v_n) \text{ with } v_i = \text{population variances of } \varepsilon_{it}$$

$$Z_m = \text{matrix of } P_m L_m^{1/2} = P_m \, diag(l_{1,}^{1/2} l_{2,}^{1/2}, ..., l_{m,}^{1/2})$$

$$\text{in which } l_i = \text{largest } m \text{ eigenvalues of correlation matrix of } \varepsilon_{it} \ \& P_m$$

$$L_m = \text{matrix of eigenvalues}$$

$$P_m = Nxm \text{ matrix of orthogonal eigenvectors}$$

Laurent, S. (2007) Estimating and Forecasting ARCH models using G@RCH, Timberlake Consultants, Ltd.,p. 177.

Orthogonal GARCH-cont'd

$f_t = (f_{1t}, f_{2t}, ..., f_{mt})$ *is a random process vector such that*

$E_{t-1}(f_t) = 0 \;\; \& \;\; Var_{t-1}(f_t) = \sum_t = (\sigma^2_{1t,}\sigma^2_{2t,}, ..., \sigma^2_{mt,}),$

$\sigma^2_{f_{it}} = (1 - \alpha_i - \beta_i) + \gamma(x_t - \bar{x}) + \alpha_i f^2_{i,t-1} + \beta_i \sigma^2_{i,t-1}$

$H_t = Var_{t-1}(\varepsilon_t) = V^{1/2} V_t V'^{1/2}$

where

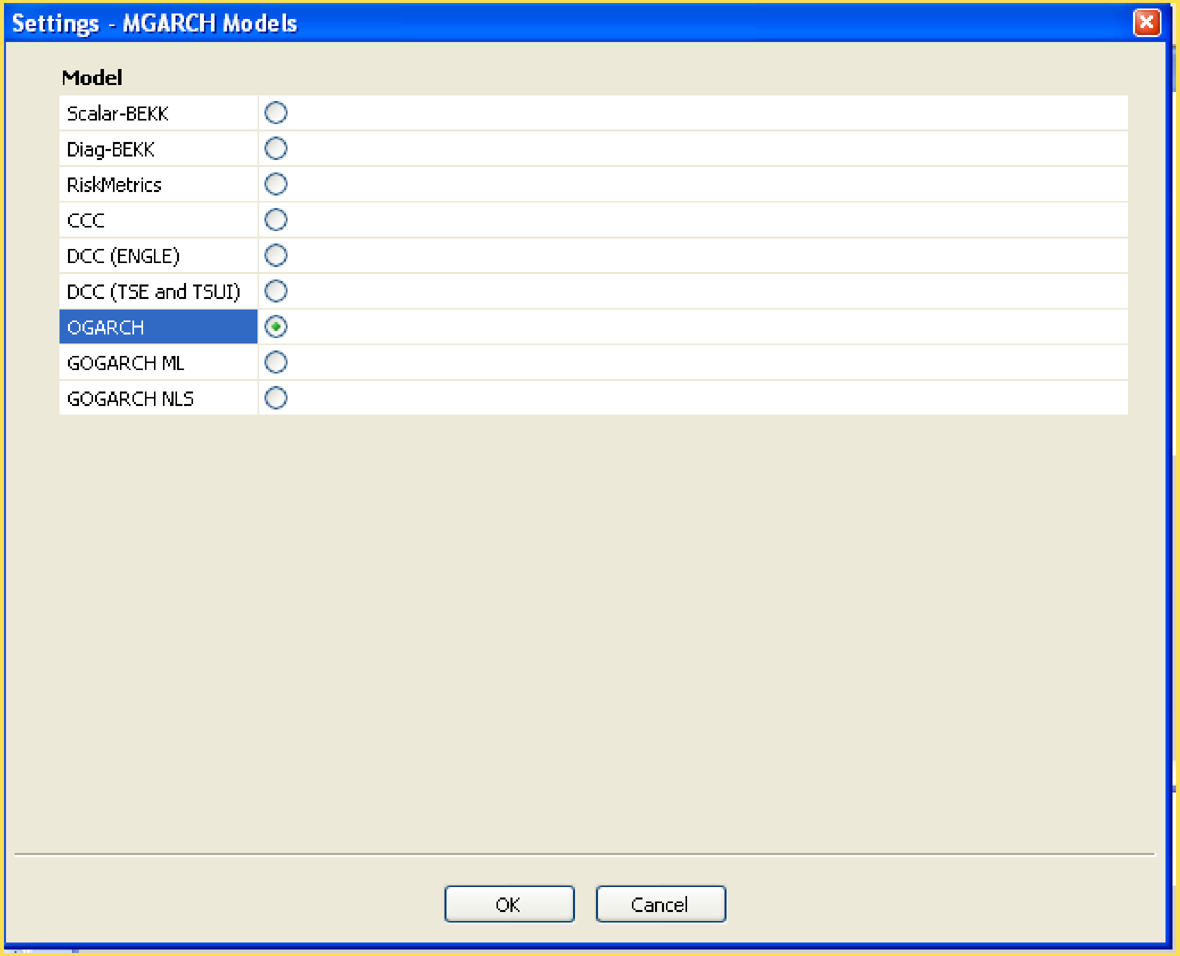
$V_t = Var_{t-1}(u_t) = Z_m \sum_t Z'_m$

Ibid, 178.

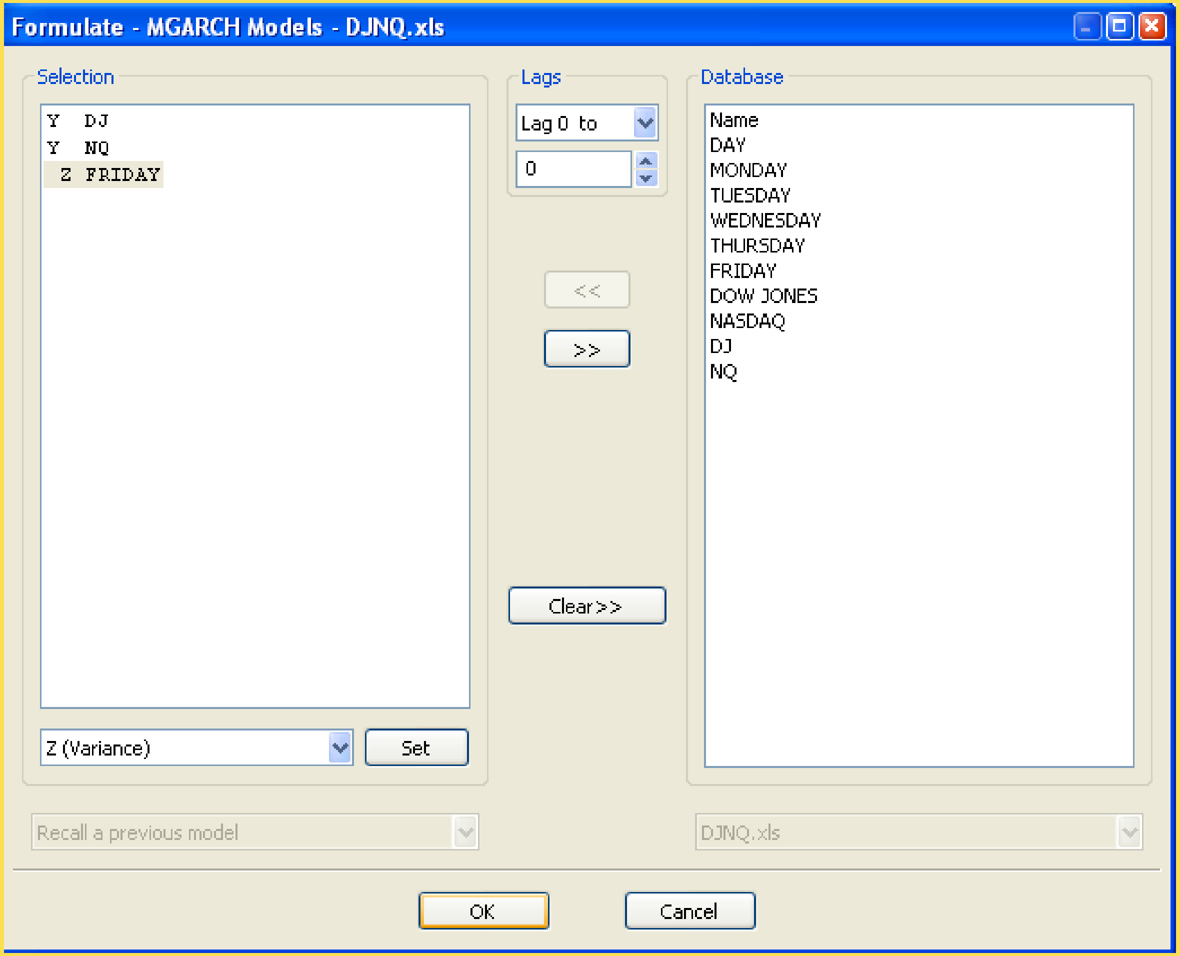
OGARCH-cont'd

- Alexander warns that high dimensional factor estimation can grind to a halt.
- She suggests low order dimensional component extraction. She extracts 2 components from 12 series.
- QMLE is used.
- ARFIMA can be specified in the mean model.

Select OGARCH

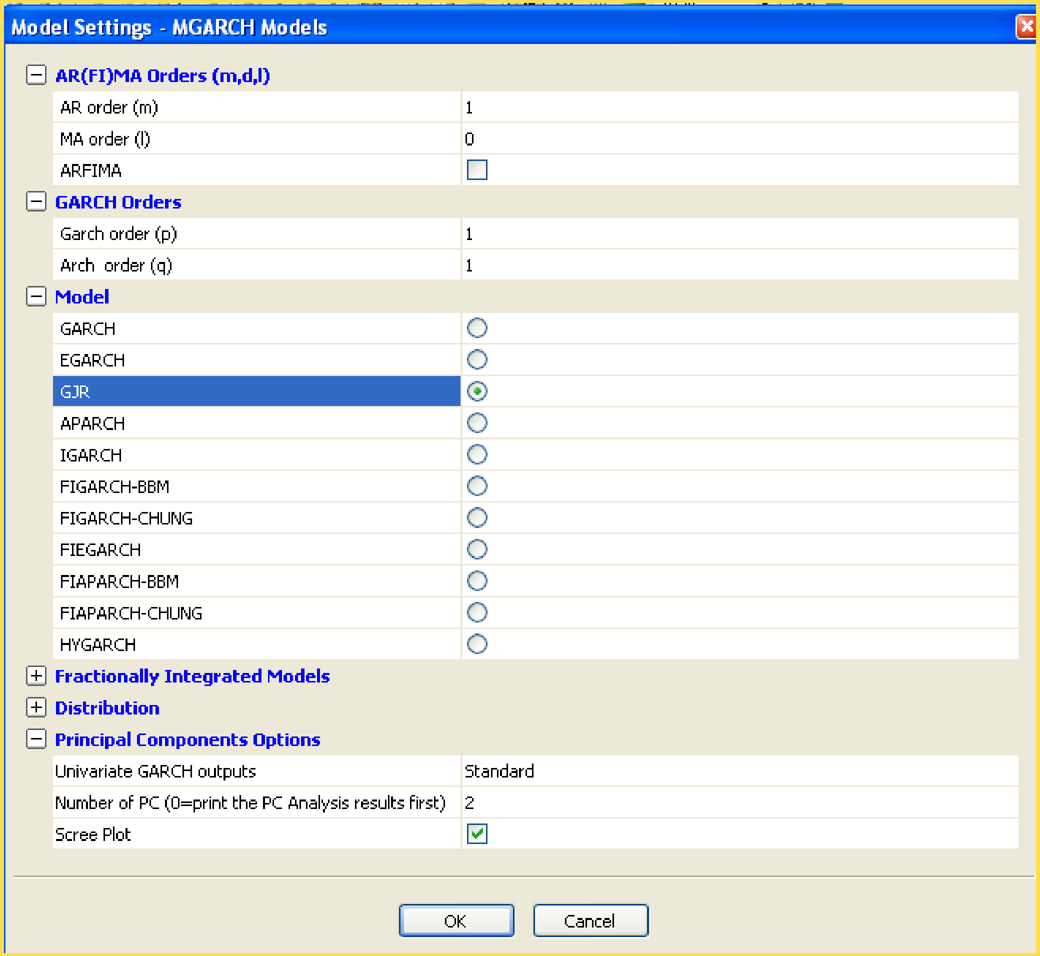


OGARCH

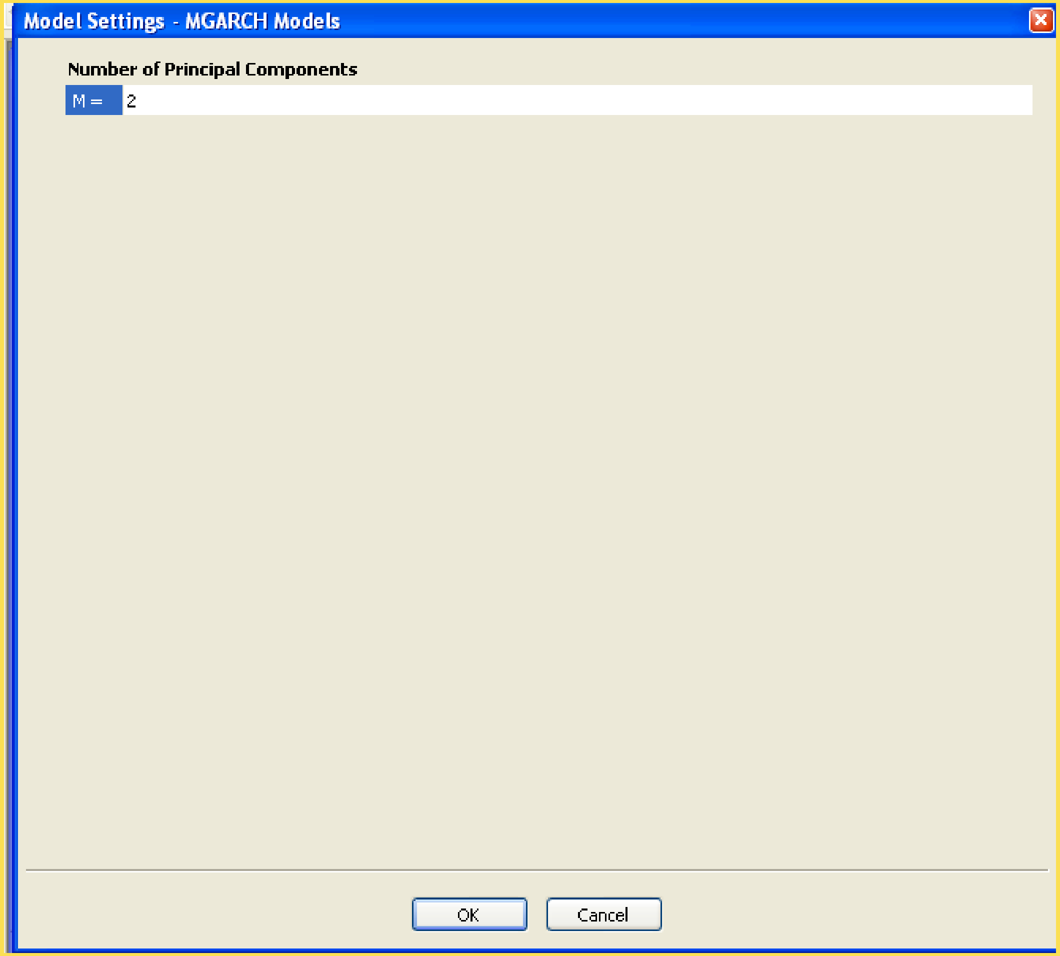


Formulate - MGARCH Models - DJNQ.xls
Selection
Y DJ
Y NQ
Z FRIDAY
Z (Variance)
Set
Lags
Lag 0 to
0
<<
>>
Clear>>
Database
Name
DAY
MONDAY
TUESDAY
WEDNESDAY
THURSDAY
FRIDAY
DOW JONES
NASDAQ
DJ
NQ
Recall a previous model
DJNQ.xls
OK
Cancel

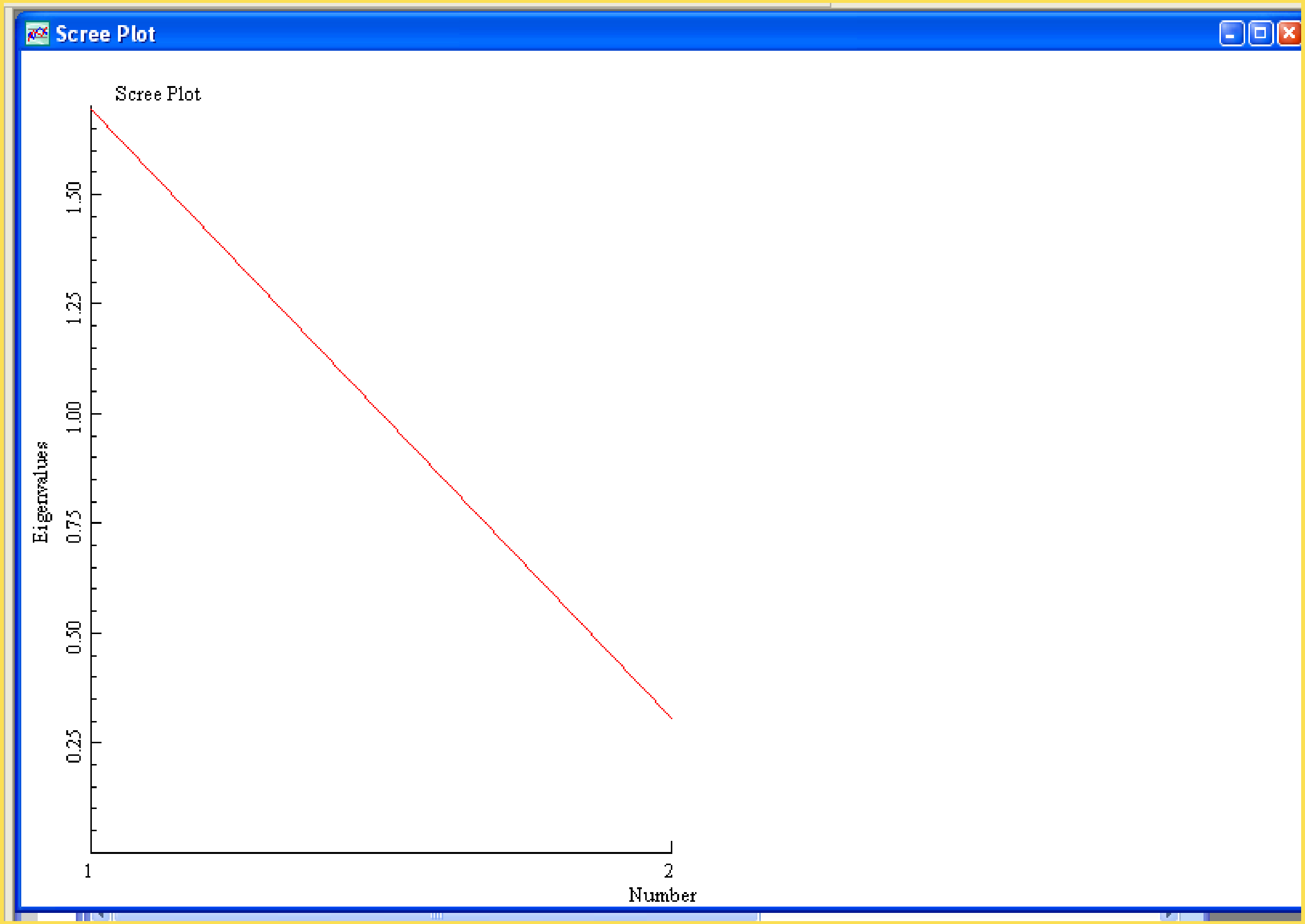
Select AR(1)-GJR-GARCH(1,1) with scree plot and standard GARCH output for 2 components



Select 2 components



Scree plot suggests 1 component



Mean model estimates

```
---------------------------Estimating the univariate GARCH model for DJ---------------------------


 ********************
 ** SPECIFICATIONS **
 ********************
Dependent variable : DJ
Mean Equation : ARMA (1, 0) model.
No regressor in the conditional mean
Variance Equation : GARCH (0, 0) model.
No regressor in the conditional variance
Normal distribution.

Strong convergence using numerical derivatives
Log-likelihood = -5596.64
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.033489   0.016156    2.073  0.0383
AR(1)               -0.001076   0.023241 -0.04630  0.9631
Cst(V)               1.022913   0.043648    23.44  0.0000

No. Observations :       3913  No. Parameters  :         3
Mean (Y)         :    0.03369  Variance (Y)    :   1.02307
Skewness (Y)     :   -0.30325  Kurtosis (Y)    :   8.12068
Log Likelihood   :  -5596.639

Estimated Parameters Vector :
 0.033489;-0.001076; 1.022913
```

Mean model estimates-cont’d

```
 ********************
 ** SPECIFICATIONS **
 ********************
Dependent variable : NQ
Mean Equation : ARMA (1, 0) model.
No regressor in the conditional mean
Variance Equation : GARCH (0, 0) model.
No regressor in the conditional variance
Normal distribution.

Strong convergence using numerical derivatives
Log-likelihood = -7248.07
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.035089   0.025376    1.383  0.1668
AR(1)                0.027975   0.026912    1.040  0.2986
Cst(V)               2.379124    0.10661    22.32  0.0000

No. Observations :      3913  No. Parameters  :         3
Mean (Y)         :   0.03527  Variance (Y)    :   2.38111
Skewness (Y)     :  -0.01238  Kurtosis (Y)    :   8.76442
Log Likelihood   : -7248.073

Estimated Parameters Vector :
 0.035089; 0.027975; 2.379124

Elapsed Time : 0.25 seconds (or 0.00416667 minutes).
```

PCA I

```
Principal Components Analysis on the Correlation matrix


    Component    Eigenvalue    Proportion    Cumulative
       1.0000        1.6929       0.84647       0.84647
       2.0000       0.30706       0.15353        1.0000

Eigenvectors

                    PC_1          PC_2
DJ              -0.70711       0.70711
NQ              -0.70711      -0.70711

Correlation between the PC and the variables

                    PC_1          PC_2
DJ              -0.92004       0.39183
NQ              -0.92004      -0.39183

STEP 1: PC Analysis
-------------------

Principal Components Analysis on the Correlation matrix


    Component    Eigenvalue    Proportion    Cumulative
       1.0000        1.6929       0.84647       0.84647
       2.0000       0.30706       0.15353        1.0000
```

PCA II

Results

```
Eigenvectors

                PC_1          PC_2
DJ          -0.70711       0.70711
NQ          -0.70711      -0.70711

Correlation between the PC and the variables

                PC_1          PC_2
DJ          -0.92004       0.39183
NQ          -0.92004      -0.39183

O-GARCH rotation matrix
-----------------------
Rotation matrix (Z_m = P_m L_m^1/2 with m=2)
     -0.92004       0.39183
     -0.92004      -0.39183



STEP 2: ML Estimation of the GARCH-type models on the unobserved factors
------------------------------------------------------------------------
```

Univariate GARCH model for PC(1)

Results

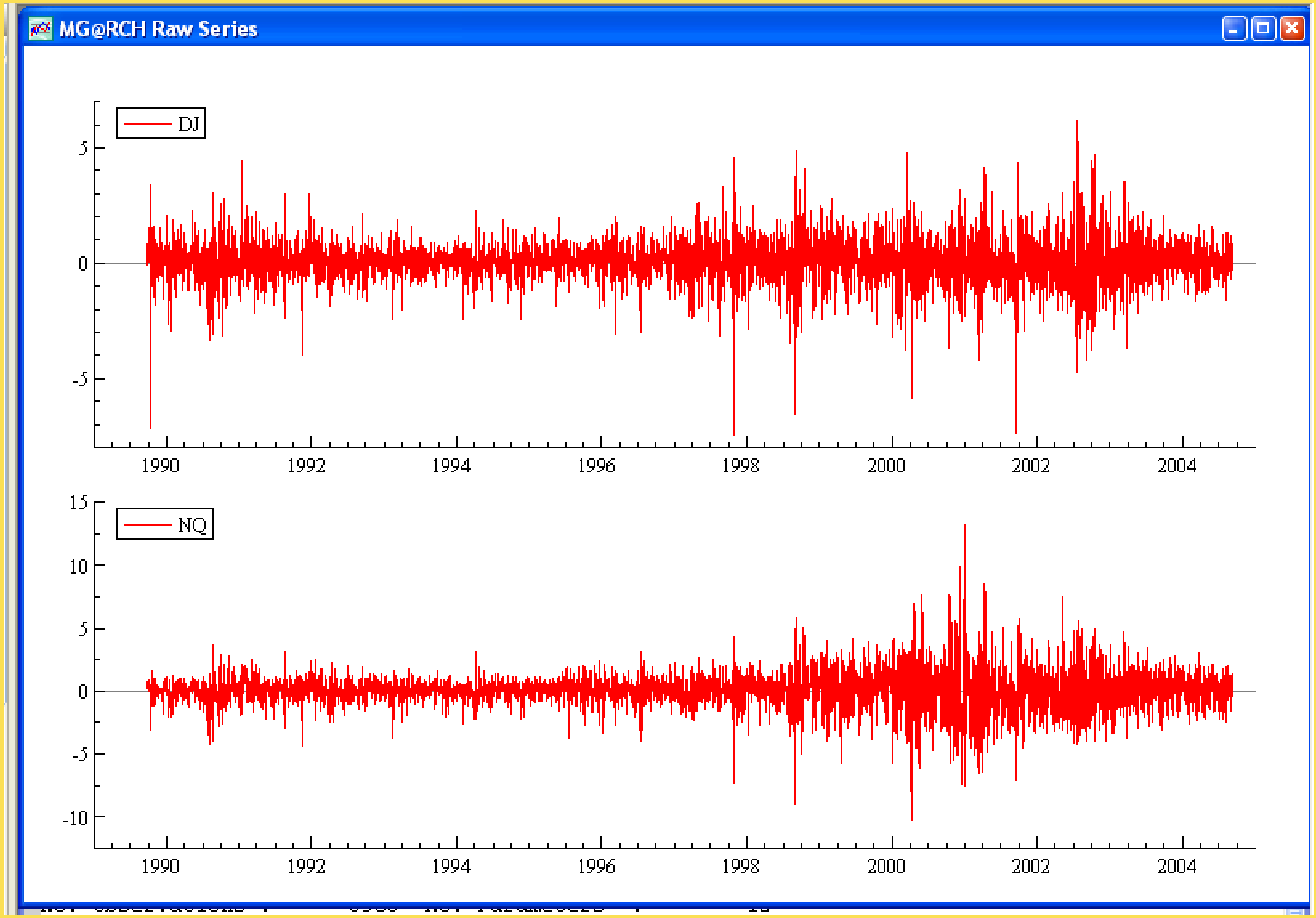
```
 ********************
 ** SPECIFICATIONS **
 ********************
Dependent variable : PC(1)
Mean Equation : ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation : GJR (1, 1) model.
            Variance Targeting
1 regressor(s) in the conditional variance.
Normal distribution.

Strong convergence using numerical derivatives
Log-likelihood = -4916.33
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
FRIDAY (V)           0.155899   0.064006    2.436  0.0149
ARCH(Alpha1)         0.112544   0.026261    4.286  0.0000
GARCH(Beta1)         0.923963   0.017442    52.97  0.0000
GJR(Gamma1)         -0.093986   0.026911   -3.492  0.0005
sigma^2             -0.020710

No. Observations :      3913  No. Parameters   :         4
Mean (Y)         :  -0.00000  Variance (Y)     :   1.00000
Skewness (Y)     :   0.15700  Kurtosis (Y)     :   7.66192
Log Likelihood   : -4916.333

The sample mean of squared residuals was used to start recursion.
Positivity & stationarity constraints are not computed because there are
explanatory variables in the conditional variance equation.

Estimated Parameters Vector :
 0.155899; 0.112544; 0.923963;-0.093986
```

Univariate GARCH model for PC(2)

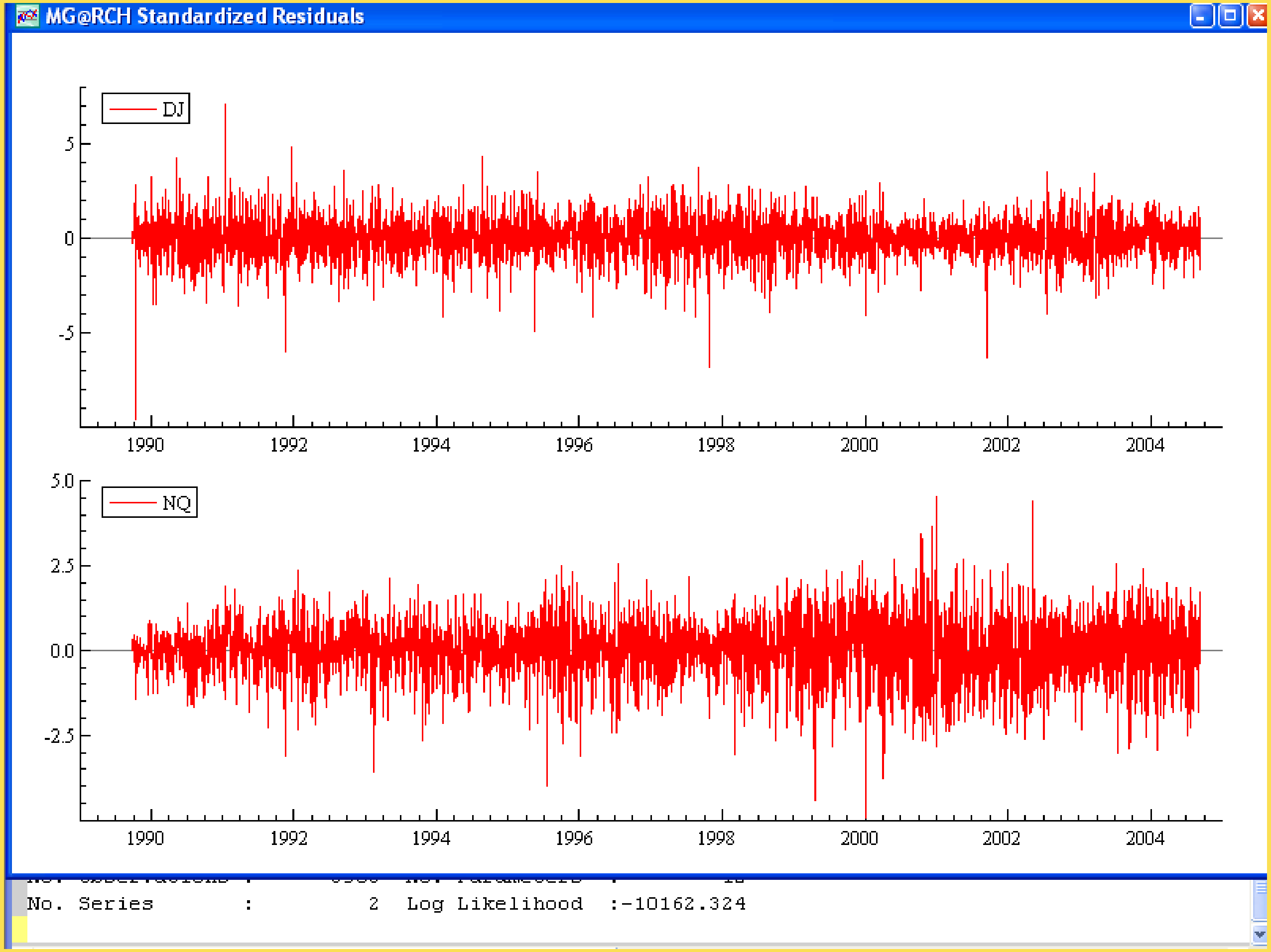
Results

```
 ** SPECIFICATIONS **
 ********************
Dependent variable : PC(2)
Mean Equation : ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation : GJR (1, 1) model.
            Variance Targeting
1 regressor(s) in the conditional variance.
Normal distribution.

Strong convergence using numerical derivatives
Log-likelihood = -4804.16
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
FRIDAY (V)           0.061350   0.053089    1.156  0.2479
ARCH(Alpha1)         0.037768   0.010111    3.735  0.0002
GARCH(Beta1)         0.952159  0.0098491    96.67  0.0000
GJR(Gamma1)          0.013421   0.010270    1.307  0.1914
sigma^2             -0.008914

No. Observations :       3913  No. Parameters   :         4
Mean (Y)         :   -0.00000  Variance (Y)     :   1.00000
Skewness (Y)     :    0.36774  Kurtosis (Y)     :  10.02964
Log Likelihood   :  -4804.160

The sample mean of squared residuals was used to start recursion.
Positivity & stationarity constraints are not computed because there are
explanatory variables in the conditional variance equation.

Estimated Parameters Vector :
 0.061350; 0.037768; 0.952159; 0.013421
```

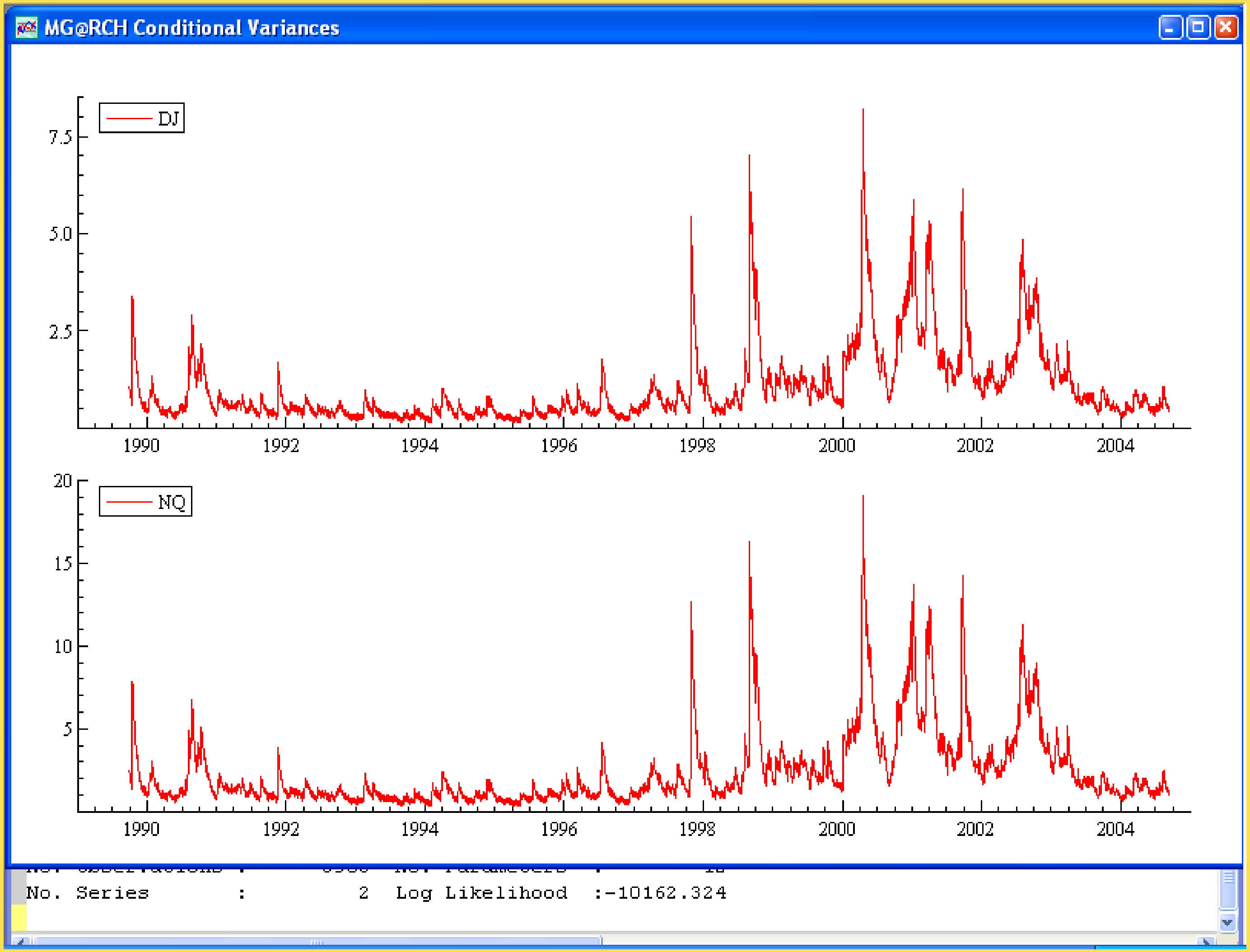
Graphs of the Raw Series



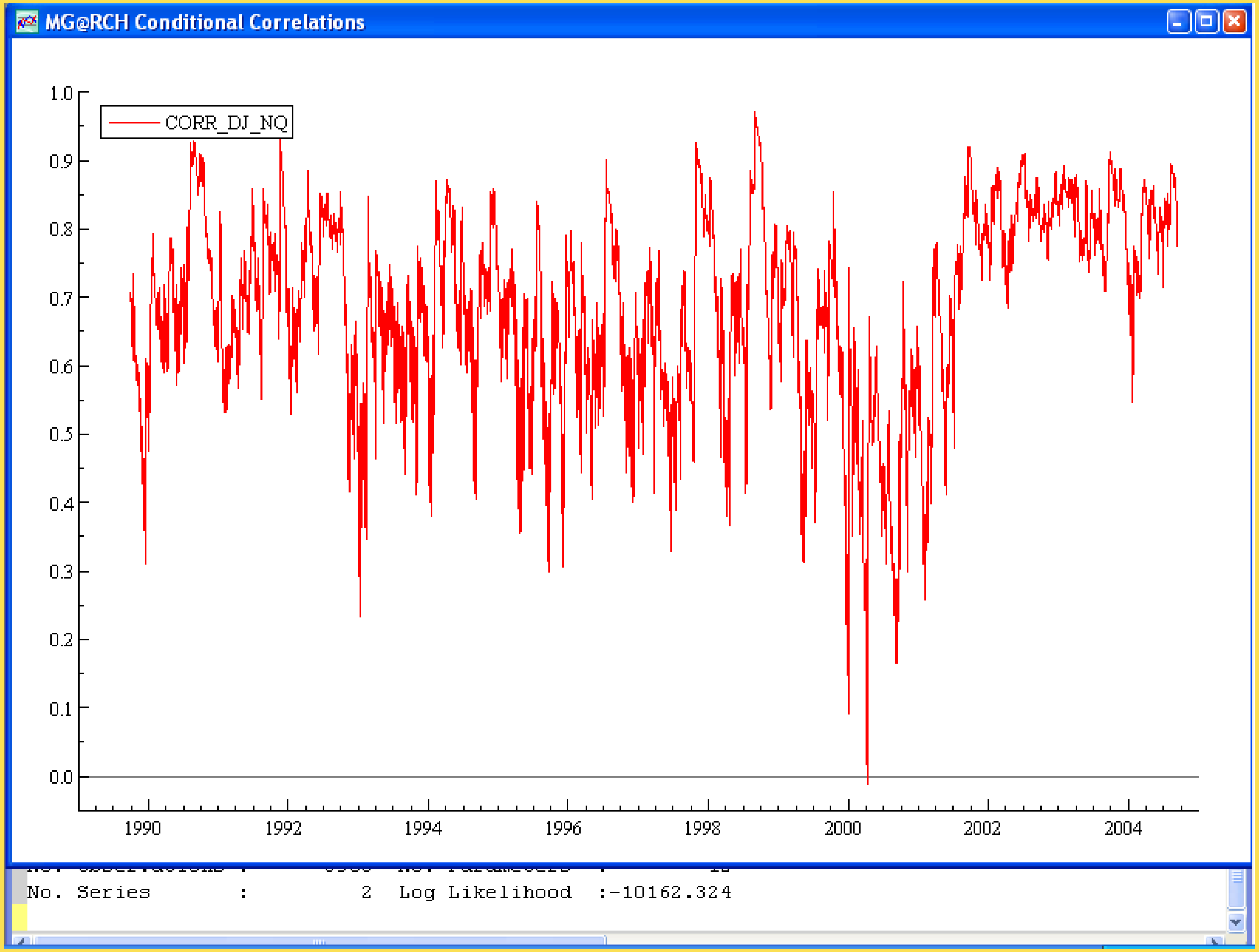
Graphs of Standardized Residuals



Conditional Variances



Graph of the Conditional Correlations



Forecasts

- Prints conditional mean forecasts
- Prints conditional variance forecasts
- Prints v-c forecasts
- Prints conditional correlation forecasts

Printed forecasts

```
Results
Conditional Mean Forecast.

Horizon          DJ          NQ
      1     0.03442     0.05863
      2      0.0344     0.03652
      3      0.0344      0.0359
      4      0.0344     0.03588
      5      0.0344     0.03588
      6      0.0344     0.03588
      7      0.0344     0.03588
      8      0.0344     0.03588
      9      0.0344     0.03588
     10      0.0344     0.03588
---------------

Conditional V-C Forecast.
step 1:
           DJ            NQ
      0.46575       0.53764
      0.53764        1.0837
step 2:
           DJ            NQ
      0.46125       0.53136
      0.53136        1.0732
step 3:
           DJ            NQ
      0.45679       0.52515
      0.52515        1.0628
step 4:
           DJ            NQ
      0.59878       0.71220
      0.71220        1.3932
step 5:
           DJ            NQ
```

Printed forecasts-cont'd.

Results

```
       0.86518        1.6673
step 10:
            DJ            NQ
       0.70956       0.85526
       0.85526        1.6510

Conditional Correlation Forecast.
step: 1
            DJ            NQ
        1.0000       0.75676
       0.75676        1.0000
step: 2
            DJ            NQ
        1.0000       0.75523
       0.75523        1.0000
step: 3
            DJ            NQ
        1.0000       0.75369
       0.75369        1.0000
step: 4
            DJ            NQ
        1.0000       0.77975
       0.77975        1.0000
step: 5
            DJ            NQ
        1.0000       0.77835
       0.77835        1.0000
step: 6
            DJ            NQ
        1.0000       0.77693
       0.77693        1.0000
step: 7
            DJ            NQ
        1.0000       0.77551
```

Multivariate tests

Results

```
  ***********
 ** TESTS **
***********
Information Criteria (to be minimized)
Akaike          5.213592  Shibata         5.213573
Schwarz         5.232868  Hannan-Quinn    5.220433
---------------

Vector Normality test:   Chi^2(4)   =   1071.9 [0.0000]**

Hosking's Multivariate Portmanteau Statistics on Standardized Residuals
  Hosking(  5) =  25.9177   [0.1324999]
  Hosking( 10) =  42.0959   [0.3384236]
  Hosking( 20) =  107.230   [0.0190009]
  Hosking( 50) =  243.977   [0.0163043]
Warning: P-values have been corrected by 1 degree of freedom
---------------

Hosking's Multivariate Portmanteau Statistics on Squared Standardized Residuals
  Hosking(  5) =  287.508   [0.0000000]
  Hosking( 10) =  509.086   [0.0000000]
  Hosking( 20) =  810.765   [0.0000000]
  Hosking( 50) =  1779.90   [0.0000000]
Warning: P-values have been corrected by 2 degrees of freedom
---------------

Li and McLeod's Multivariate Portmanteau Statistics on Standardized Residuals
  Li-McLeod(  5) =  25.9187   [0.1324717]
  Li-McLeod( 10) =  42.1044   [0.3380914]
  Li-McLeod( 20) =  107.152   [0.0192378]
  Li-McLeod( 50) =  243.785   [0.0166497]
Warning: P-values have been corrected by 1 degree of freedom
---------------
```

Generalized OGARCH (van der Weide, 2002)

- One can test whether the correlations between the components are really zero.
- This model outperforms the OGARCH sometimes, generating a log-likelihood may be lower.
- The orthogonality assumption between OGARCH components is relaxed. Rather the Z matrix in

$$u_t = Zf_t$$

- is assumed to be square and invertible only.
- (Laurent, 2007, class notes).

GOGARCH - (Laurent notes, con'td)

where P and L are defined as the eigenvectors and eigenvalues,

$$\mathrm{m} = \mathrm{N},\ \ \mathrm{Z_m} = \mathrm{Z} = \mathrm{PL}^{1/2}\mathrm{U}$$

and

U is the product of N(N-1)/2 rotation matrices:

$$U = \prod_{i<j} G_{ij}(\delta_{ij}),\ \ -\pi \le \delta_{ij} \le \pi\,,\ \ i, j = 1,2,\ldots,n$$

Generalized Orthogonal GARCH

$$R_t = J_t^{-1} V_t J_t^{-1}$$

where

$$R_t = \text{implied correlation matrix}$$

$$J_t = (V_t \text{ e } I_m)^{1/2} \quad \text{and} \quad V_t = Z \Sigma Z'$$

$$\text{e} = \text{Hadamard (element by element) product}$$

Specification tests (Laurent notes cont'd).

- The specification tests (univariate and multivariate) are used to assess the fit and specification of the model.
- Univariate tests are applied to each u_{it}
- Univariate tests are applied to each z_{it}. Univariate tests are applied to each $u_{it}u_{jt}$ to assess the covariance specification.
- Multivariate tests are applied to the vector z_t as a whole.

Rotation matrices

For the trivariate case, the rotation matrices are

$$G_{12} = \begin{pmatrix} \cos\delta_{12} & \sin\delta_{12} & 0 \\ -\sin\delta_{12} & \cos\delta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad G_{13} = \begin{pmatrix} \cos\delta_{12} & \sin\delta_{12} & 0 \\ 0 & 1 & 0 \\ -\sin\delta_{13} & \cos\delta_{13} & 0 \end{pmatrix}$$

There are N(N-1)/2 rotation angles are the parameters to be Estimated.

Conditional Correlation Models

- Bollerslev(1990) introduced constant conditional correlation estimator.
- The Dynamic conditional Correlation between the conditional variances is made time-varying by Engle.
- Forecasts are possible
- Graphs of conditional correlations are possible
- Application: Better for computing time-varying hedge ratios than a linear regression model.
- Takes into account conditional heteroskedasticity in the spot market.

Conditional Correlation Models

- Bollerslev's (1990) Constant Conditional Correlation
- Tse and Tsui(2002) Dynamic Conditional Correlation
- Engle(2002) Dynamic Conditional Correlation

Constant Conditional correlation (Bollerslev, 1990)

- Two or more univariate GARCH models are estimated.
- Nonlinear combinations of conditional variances from different GARCH models

$$H_t = D_t R D_t = (\rho_{ij}\sqrt{h_{iit}h_{jjt}})$$

$$where$$

$$D_t = diag(h_{11t}^{1/2}, h_{22t}^{1/2}, \ldots, h_{NNt}^{1/2})$$

$$h_{iit} = any\ univariate\ GARCH\ model$$

$$R = \rho_{ij}\ (a\ symmetric\ positive\ definite\ matrix\ with\ \rho_{ij} = 1,\ \forall i$$

Constant conditional correlation-cont'd

Originally, the CCC model had a GARCH(1,1) specification for each

Conditional variance in $D_{t:}$

$$h_{iit} = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{ii,t-1}$$

The CCC model has N(N+5)/2 parameters. H_t is positive definite if and only if all N conditional variances are positive and R is positive definite. The unconditional variances are easy to obtain but unconditional covariances are difficult to calculate because of nonlinearity (Laurent,S. G@RCH manual 192).

Dynamic Conditional Correlations: A new class of MGARCH

- "Rob Engle (1999) in "Dynamic Conditional Correlation—A Simple Class of Multivariate GARCH Modles, has written that, "Time-varying correlations are often estimated with MGARCH that are linear in their squares and cross-products."
- "They have flexibility of univariate GARCH models…."
- "They do not have the complexity of MGARCCH."
- "They have parsimonious parametric models for the correlations."
- They perform well in a variety of situations and provide sensible empirical results."

DCC models

- Advantages:
  - The number of parameters to be estimated is independent of the number of series to be correlated.
  - Potentially very large correlation matrices can be estimated.
  - The rolling correlation estimator can be computed.

Dynamic Conditional Correlation (Tse and Tsui, 2002)

$$H_t = D_t R_t D_t$$

where

$$D_t = diag(h_{11t}^{1/2}, h_{22t}^{1/2}, \ldots, h_{NNt}^{1/2})$$

$$h_{iit} = \text{any univariate GARCH model}$$

so that

$$R_t = (1 - \theta_1 - \theta_2)\bar{R} + \theta_1 \psi_{t-1} + \theta_2 R_{t-1}$$

where R = *a symmetric positive definite parameter matrix*

with $\rho_{ii} = 1$, *and*

$$\psi_{t-1} = \frac{\sum_{m=1}^{M} \varepsilon_{i,t-m} \varepsilon_{j,t-m}}{\sqrt{\left(\sum_{m=1}^{M} \varepsilon_{i,t-m}^2\right)\left(\sum_{m=1}^{M} \varepsilon_{j,t-m}^2\right)}}$$

Tse and Tsui's Dynamic Conditional Correlation make R time dependent

$$R_t = (1-\theta_1-\theta_2)\bar{R} + \theta_1\psi_{t-1} + \theta_2 R_{t-1}$$

$where \;\; R = a \; symmetric \; positive \; definite \;\; parameter \;\; matrix$

$with \;\; \rho_{ii} = 1, \; and$

$$\psi_{t-1} = \frac{\sum_{m=1}^{M}\varepsilon_{i,t-m}\varepsilon_{j,t-m}}{\sqrt{\left(\sum_{m=1}^{M}\varepsilon_{i,t-m}^2\right)\left(\sum_{m=1}^{M}\varepsilon_{j,t-m}^2\right)}} = B_{t-1}^{-1}L_{t-1}L_{t-1}^{'}B_{t-1}^{-1}$$

$where \;\; \varepsilon_{it} = \frac{e_{it}}{\sqrt{h_{iit}}} \;\; and \;\; B_{t-1} = NxN \;\; diagonal \;\; matrix \; with \; i-th \;\; diagonal$

$element \; given \; by \;\; \sqrt{\left(\sum_{m=1}^{M}\varepsilon_{i,t-h}^2\right)} \;\; and \;\; L_{t-1} = (\varepsilon_{t-1,...,} \; \varepsilon_{t-m}) = NxN \; with$

$\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t}, ..., \varepsilon_{Nt})'$

Tse and Tsui's DCC

$$\bar{R} = \frac{1}{T}\sum_{t=1}^{T}\varepsilon_t \varepsilon_t '$$

$Only\ if\ \ R_t\ has\ M \leq N\ order\ will\ \psi_{t-1}\ be\,assured\ of$
$positivity.$

Dynamic Conditional Correlation (Engle, 2002)

Tse and Tsui

$$\rho_{12t} = (1-\theta_t - \theta_2)\rho_{12} + \theta_2 \rho_{12,t-1} + \theta_1 \frac{\sum_{m=1}^{M} \varepsilon_{i,t-m}\varepsilon_{j,t-m}}{\sqrt{\left(\sum_{m=1}^{M} \varepsilon_{i,t-m}^2\right)\left(\sum_{m=1}^{M} \varepsilon_{j,t-m}^2\right)}}$$

Covariance

Engle

$$\rho_{12t} = \frac{(1-\alpha-\beta)\bar{q}_{12} + \alpha\varepsilon_{1,t-1}\varepsilon_{2,t-1} + \beta q_{12,t-1}}{\sqrt{\left((1-\alpha-\beta)\bar{q}_{11} + \alpha\varepsilon_{1,t-1}^2 + \beta q_{11,t-1}\right)\left((1-\alpha-\beta)\bar{q}_{22} + \alpha\varepsilon_{2,t-1}^2 + \beta q_{22,t-1}\right)}}$$

Std devs

Engle's DCC

$$R_t = diag(q_{11,t,}^{-1/2} \ldots, \; q_{NN,t}^{-1/2}) Q_t diag(q_{11,t,}^{-1/2} \ldots, \; q_{NN,t}^{-1/2}),$$

$where \; Q_t = (q_{11,t}^{-1/2}) \; is \; an \; NxN \; symmetric \; positive \; definite$

$matrix \; given \; by$

$$Q_t = \bar{R}\left(1 - \alpha - \beta\right) + \alpha\left(\varepsilon_{t-1}\varepsilon'_{t-1}\right) + \beta Q_{t-1}$$

Equation borrowed from Rob Engle's presentation
on DCC, ISF2007.

Parsimony prevails

- If the individual processes are GARCH(1,1), the DCC has only (N+1)(N+4)/2 parameters.

Two-step Quasi-Maximum Likelihood Estimation

- Engle and Sheppard (2001) show that in the DCC case, the log-likelihood can be written as the sum of the mean and volatility part.
- Step 1and 2: QML function corresponds to the sum of the LL functions of N univariate models.

$$QL1_t(\theta_1^*) = -\frac{1}{2}\sum_{t=1}^{T}\sum_{i=1}^{N}\left[\log(h_{iit}) + \frac{(y_{it}-\mu_{it})^2}{h_{iit}}\right]$$

$$\textit{Given } \theta_1^* \textit{ a consistent though inefficient estimator of } \theta_2^* \textit{ comes from}$$

$$\max imizing:$$

$$QL1_t(\theta_2^*) = -\frac{1}{2}\sum_{t=1}^{T}\left(\log|Rt| + \mu_t^{'} R^{-1}\mu_t\right)$$

$$\textit{where } \mu_t = D_t^{-1}(y_{it}-\mu_{it})$$

Engle DCC output

```
---------------------------Estimating the univariate GARCH model for DJ---------------------------


 ********************
 ** SPECIFICATIONS **
 ********************
Dependent variable : DJ
Mean Equation : ARMA (1, 0) model.
No regressor in the conditional mean
Variance Equation : GJR (1, 1) model.
No regressor in the conditional variance
Normal distribution.

Weak convergence (no improvement in line search) using numerical derivatives
Log-likelihood = -5150.9
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.029764   0.013591    2.190  0.0286
AR(1)                0.021967   0.017049    1.288  0.1977
Cst(V)               0.011930  0.0047754    2.498  0.0125
ARCH(Alpha1)         0.008823  0.0068322    1.291  0.1966
GARCH(Beta1)         0.936954   0.016569    56.55  0.0000
GJR(Gamma1)          0.081739   0.024269    3.368  0.0008

No. Observations :       3913  No. Parameters   :         6
Mean (Y)         :    0.03369  Variance (Y)     :   1.02307
Skewness (Y)     :   -0.30325  Kurtosis (Y)     :   8.12068
Log Likelihood   :  -5150.903

The sample mean of squared residuals was used to start recursion.
The condition for existence of the second moment of the GJR is observed.
This condition is alpha(1) + beta(1) + k gamma(1) < 1 (with k = 0.5 with this distribution.)
In this estimation, this sum equals 0.986647.
The condition for existence of the fourth moment of the GJR is observed.
```

Engle’s DCC

```
--------------------------Estimating the univariate GARCH model for NQ--------------------------


 ********************
 ** SPECIFICATIONS **
 ********************
Dependent variable : NQ
Mean Equation : ARMA (1, 0) model.
No regressor in the conditional mean
Variance Equation : GJR (1, 1) model.
No regressor in the conditional variance
Normal distribution.

Strong convergence using numerical derivatives
Log-likelihood = -6245.8
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.044543   0.017815    2.500  0.0125
AR(1)                0.110349   0.017202    6.415  0.0000
Cst(V)               0.015383  0.0059041    2.605  0.0092
ARCH(Alpha1)         0.039653   0.010897    3.639  0.0003
GARCH(Beta1)         0.912374   0.018410    49.56  0.0000
GJR(Gamma1)          0.079668   0.024063    3.311  0.0009

No. Observations :       3913  No. Parameters   :         6
Mean (Y)         :    0.03527  Variance (Y)     :   2.38111
Skewness (Y)     :   -0.01238  Kurtosis (Y)     :   8.76442
Log Likelihood   :  -6245.795

The sample mean of squared residuals was used to start recursion.
The condition for existence of the second moment of the GJR is observed.
This condition is alpha(1) + beta(1) + k gamma(1) < 1 (with k = 0.5 with this distribution.)
In this estimation, this sum equals 0.99186.
```

Engle’s Dynamic Conditional Correlation

```
 *************
 ** SERIES  **
*************

       #
DJ     1
NQ     2

 ******************************
 ** MG@RCH( 2) SPECIFICATIONS **
 ******************************
Conditional Variance : Dynamic Correlation Model (Engle)
Multivariate Student distribution, with 8.25629 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = -9659.62
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
alpha                0.044075  0.0062293    7.075  0.0000
beta                 0.945047  0.0087833    107.6  0.0000
df                   8.256294    0.70803    11.66  0.0000

Unconditional Correlation (CCC)
rho_21               0.692362
No. Observations :      3913  No. Parameters   :        16
No. Series       :         2  Log Likelihood   : -9659.624
Elapsed Time : 0.172 seconds (or 0.00286667 minutes).
```

Alpha

Beta

df

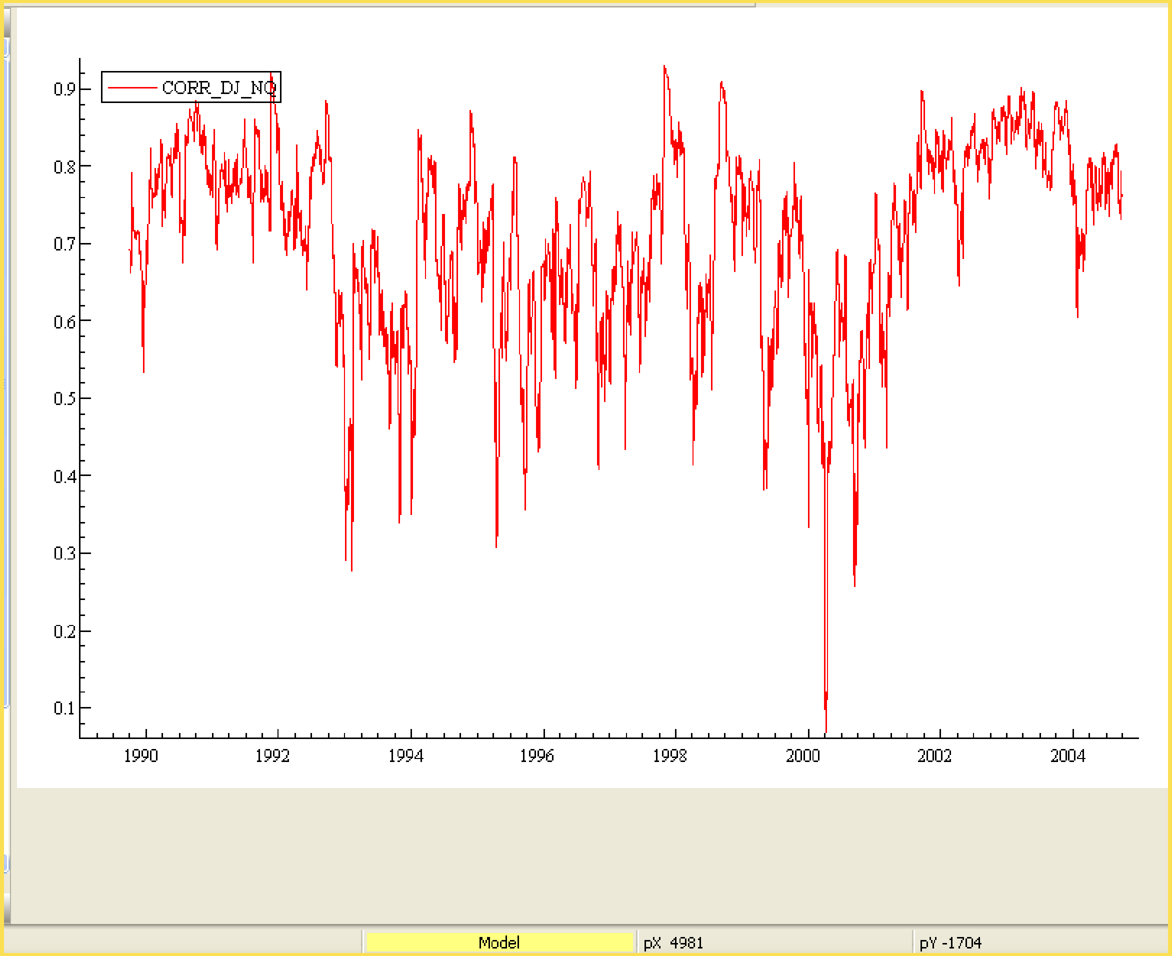
Forecasts

```
Conditional Mean Forecast.

Horizon        DJ          NQ
      1   0.01624    -0.07605
      2   0.02947     0.03124
      3   0.02976     0.04307
      4   0.02976     0.04438
      5   0.02976     0.04453
      6   0.02976     0.04454
      7   0.02976     0.04454
      8   0.02976     0.04454
      9   0.02976     0.04454
     10   0.02976     0.04454
---------------

Conditional V-C Forecast.
step 1:
           DJ          NQ
      0.61686     0.64156
      0.64156      1.1277
step 2:
           DJ          NQ
      0.62055     0.64441
      0.64441      1.1339
step 3:
           DJ          NQ
      0.62420     0.64723
      0.64723      1.1401
step 4:
           DJ          NQ
      0.62779     0.65002
      0.65002      1.1462
step 5:
           DJ          NQ
      0.63134     0.65277
[illegible]
```

Forecasts of conditional correlation

```
Conditional Correlation Forecast.
step: 1
            DJ            NQ
        1.0000       0.76920
       0.76920        1.0000
step: 2
            DJ            NQ
        1.0000       0.76820
       0.76820        1.0000
step: 3
            DJ            NQ
        1.0000       0.76723
       0.76723        1.0000
step: 4
            DJ            NQ
        1.0000       0.76628
       0.76628        1.0000
step: 5
            DJ            NQ
        1.0000       0.76534
       0.76534        1.0000
step: 6
            DJ            NQ
        1.0000       0.76443
       0.76443        1.0000
step: 7
            DJ            NQ
        1.0000       0.76354
       0.76354        1.0000
step: 8
            DJ            NQ
        1.0000       0.76266
       0.76266        1.0000
step: 9
            DJ            NQ
```

Graphical Conditional Correlation



Diagnostic Tests for Conditional Correlations

- Testing for misspecification of the conditional mean or variance equation:
- Hosking's (1980) Multivariate Box-Ljung Q statistics:

$$Hosking(1980)(m) = T^2 \sum_{j=1}^{m} (T-J)^{-1} tr\{C_{y_t}^{-1}(0) C_{y_t}(j) C_{y_t}^{-1}(0) C_{y_t}^{'}(j)\}$$

$where$

$y_t = vector\ of\ observed\ returns$

$C_{y_t}(j) = sample\ autocovariance\ matrix\ of\ order\ j$

$H_0 : no\ serial\ correlation$

Testing Misspecification in mean model

- Qing Li and Dennis McLeod's(1981) Multivariate Portmanteau test of residuals and squared residuals (Li, W.K.(2004) Diagnostic Checks in Time Series, p.10)

$$Q_m^* = Q_m + \frac{k^2 m(m+1)}{2m} \sim \chi^2 \ with \ df = k^2(m-s)$$

where

$$Q_m = n\sum_{k=1}^{m} \hat{r}_k^2 = Box-Pierce \ statistic,$$

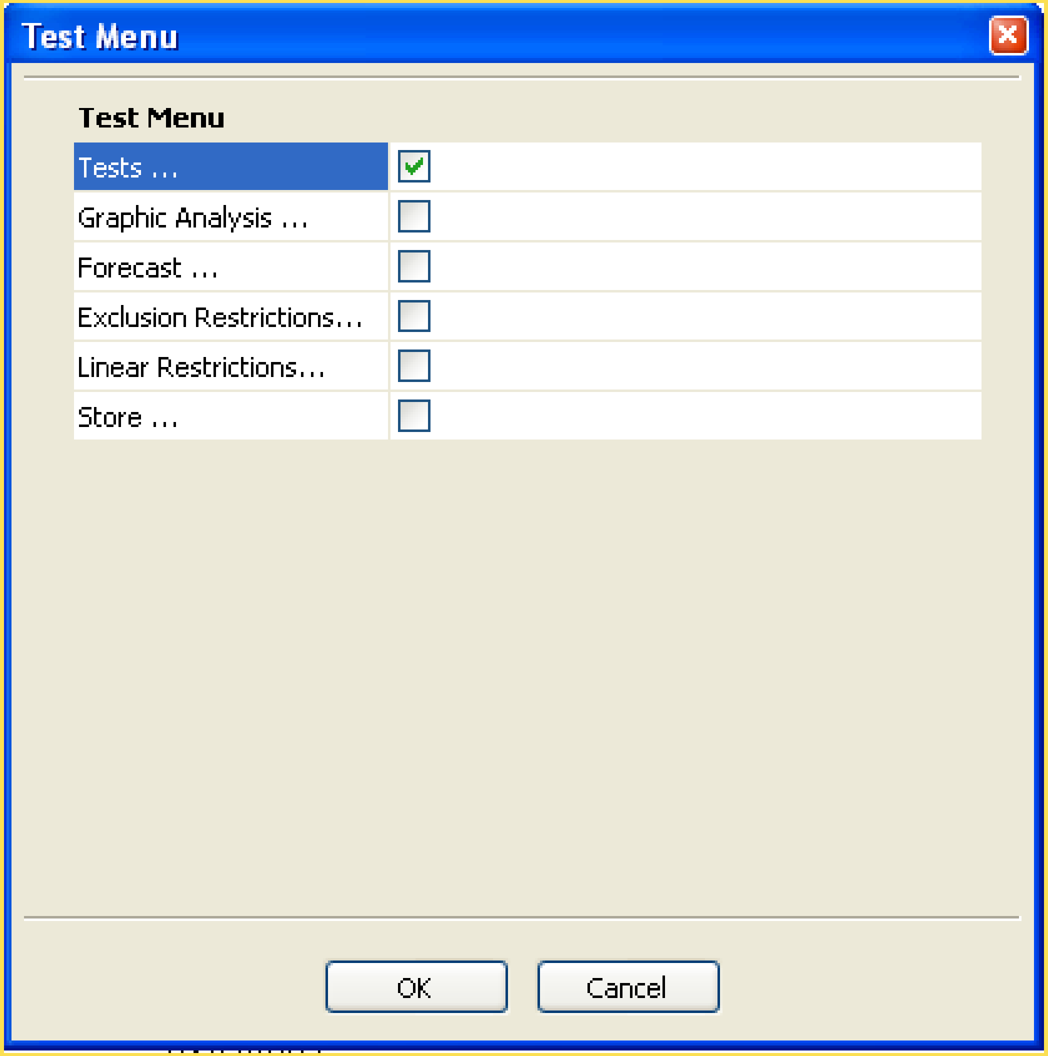
$m = lag \ order,$

$k = N$

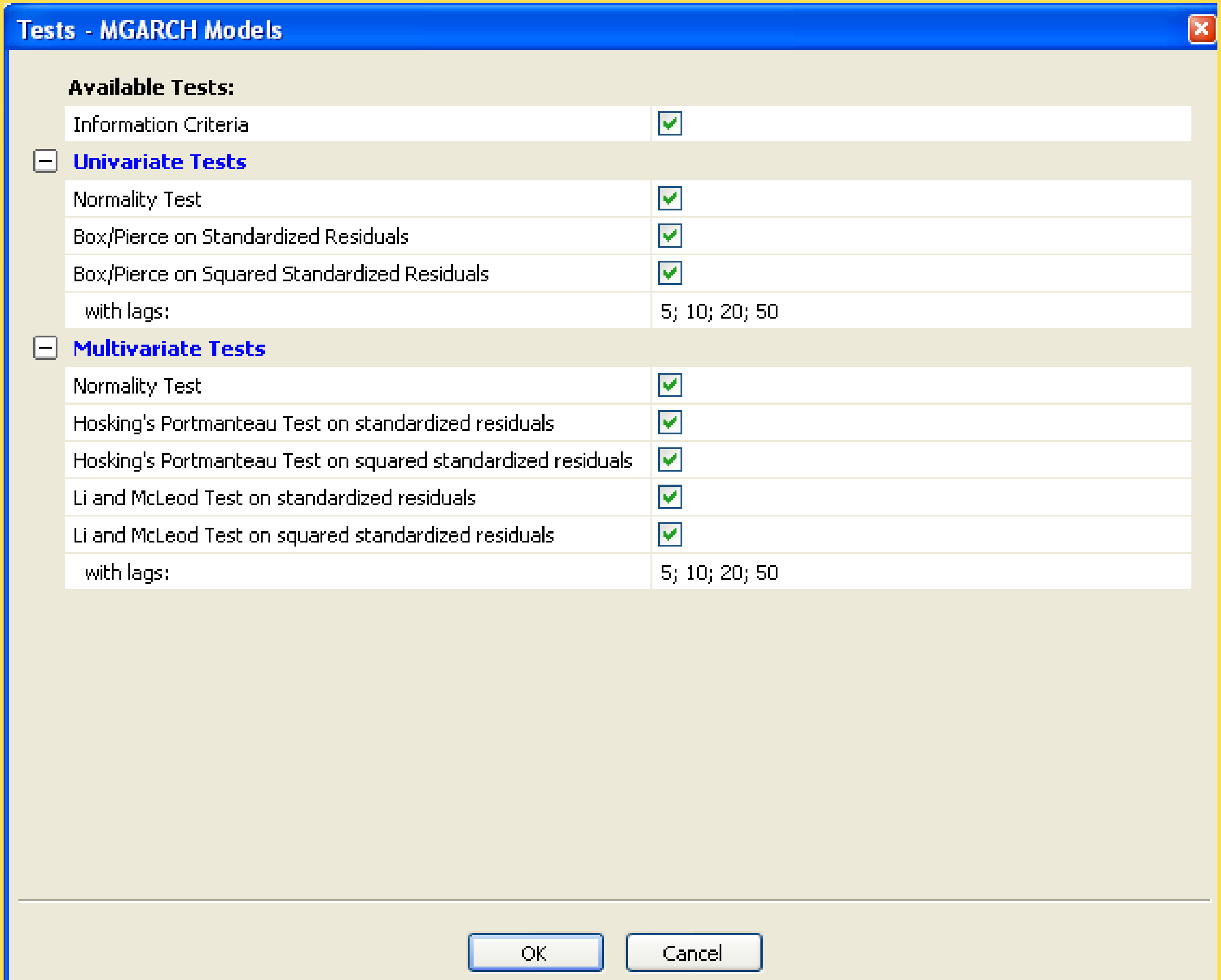
$s = p + q \ \ from \ ARMA(p,q) \ orders$

This test is applied to z^2 *to test misspecification in variance model.*

We opt for the tests in the Test Menu



Select both univariate and multivariate tests



Univariate test output

```
Individual Normality Tests
--------------------------

 Series: DJ

                   Statistic        t-Test       P-Value
Skewness            -0.20445        5.2084   1.9044e-007
Excess Kurtosis       2.0549        26.182   4.3002e-151
Jarque-Bera           711.71          .NaN   2.8505e-155

 Series: NQ

                   Statistic        t-Test       P-Value
Skewness            -0.28676        7.3055   2.7631e-013
Excess Kurtosis      0.77813        9.9141   3.6133e-023
Jarque-Bera           151.49          .NaN   1.2698e-033

Q-Statistics on Standardized Residuals

 Series: DJ
  Q(  5) =   2.36824    [0.7961946]
  Q( 10) =   4.02344    [0.9462835]
  Q( 20) =   14.1554    [0.8225299]
  Q( 50) =   48.0901    [0.5503445]

 Series: NQ
  Q(  5) =   4.52248    [0.4768815]
  Q( 10) =   6.28587    [0.7907014]
  Q( 20) =   29.9389    [0.0708505]
  Q( 50) =   57.8160    [0.2089322]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------
```

Multivariate test output

```
Vector Normality test:   Chi^2(4)   =   470.70 [0.0000]**

Hosking's Multivariate Portmanteau Statistics on Standardized Residuals
  Hosking(  5) =  19.2460    [0.3768221]
  Hosking( 10) =  32.8050    [0.7081488]
  Hosking( 20) =  96.0638    [0.0807449]
  Hosking( 50) =  241.289    [0.0193504]
Warning: P-values have been corrected by 2 degrees of freedom
---------------

Hosking's Multivariate Portmanteau Statistics on Squared Standardized Residuals
  Hosking(  5) =  29.9095    [0.0383354]
  Hosking( 10) =  48.7847    [0.1129106]
  Hosking( 20) =  91.0632    [0.1479374]
  Hosking( 50) =  202.751    [0.3934625]
Warning: P-values have been corrected by 2 degrees of freedom
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Li and McLeod's Multivariate Portmanteau Statistics on Standardized Residuals
  Li-McLeod(  5) =  19.2488    [0.3766544]
  Li-McLeod( 10) =  32.8200    [0.7074994]
  Li-McLeod( 20) =  95.9928    [0.0814885]
  Li-McLeod( 50) =  241.016    [0.0199243]
Warning: P-values have been corrected by 2 degrees of freedom
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Li and McLeod's Multivariate Portmanteau Statistics on Squared Standardized Residuals
  Li-McLeod(  5) =  29.9049    [0.0383810]
  Li-McLeod( 10) =  48.7843    [0.1129181]
  Li-McLeod( 20) =  91.0619    [0.1479583]
  Li-McLeod( 50) =  202.804    [0.3924522]
Warning: P-values have been corrected by 2 degrees of freedom
---------------
```

Recapitulation of New Features

- Autometrics
  - Automatic variable and model selection
  - Outlier and level shift detection and modeling
  - For univariate and multivariate models
- G@RCH
  - Wide variety of vanilla GARCH
  - VaR backtesting
    - Kupiec tests
    - Dynamic Quantile regression
    - Expected shortfall
  - Wide variety of Long-Memory GARCH
  - Ox Code is generated from ALT-O
  - Diffusion modeling for continuous time analysis
  - Simulated confidence intervals are CEV forecasts
  - Multivariate GARCH
  - Conditional Correlations